Employment and Wage Formation in a Growth Model with Public Capital*

Xavier Raurich
Universitat de Girona

Valeri Sorolla†
Universitat Autònoma de Barcelona

Abstract

We develop an endogenous growth model with a non-competitive labor market characterized by a monopoly union in order to study the relation between growth and employment. We show that if there is wage inertia, economic growth positively affects employment in the long run. We also use the model to analyze the effects on employment and growth of increasing public capital.

Keywords: Employment, endogenous growth, wage formation, public capital.

JEL number: E24, O41.

*We thank Jordi Caballé for helpful discussions. Raurich is grateful to Spanish Ministry of Science and Technology through DGICYT grant SEC2003-0036. Sorolla is grateful for financial support to Spanish Ministry of Science and Technology through DGICYT grant SEC2003-0036 and to Generalitat de Catalunya through grant SGR2001-164.

†Correspondence Address: Valeri Sorolla, Universitat Autònoma de Barcelona, Departament d’Economia i d’Història Econòmica, Edifici B, 08193 Bellaterra (Barcelona), Spain. Phone: (34)-935812728. e-mail: valeri.sorolla@uab.es
1. Introduction

In this paper we study the relation between growth and employment in the long run. As Aricó (2003) notes in his survey about growth and unemployment, there are a few contributions on this relationship. In Pissarides (1990) appears the first model dealing together with growth and unemployment. This author, using a matching model of the labor market, finds a positive effect of growth on employment via the capitalization effect.\(^1\) Aghion and Howitt (1994) adds to this positive effect a negative one due to the creative destruction effect of growth.

In contrast with this literature, that analyzes the relationship between growth and employment when growth is driven by technology improvement, we show the relationship between these two variables in a model where growth is driven by the accumulation of capital. Thus, in this paper, we present an alternative explanation for a long run positive effect of GDP growth on employment, which is based on real wage inertia.\(^2\) This positive effect occurs because, due to the wage inertia, economic growth do not fully translate into wage increases that prevent employment growth.

Blanchard and Wolfers (2000) argue that wage inertia explains that a decline in economic growth results in a temporary reduction in the employment rate, as delays in wage adjustment makes wages grow in excess of productivity growth during the transition.\(^3\) In contrast, we show that, in a growth model with wage inertia, a decline in the growth rate reduces the employment rate permanently, as there is a permanent wage adjustment due to sustained growth.

In this paper, wage inertia is introduced in a labor market characterized by a monopoly union that sets the wage as a mark-up over a reservation wage, that depends on past wages and the unemployment benefit. This assumption is supported by empirical evidence, which shows that there is strong wage inertia in the wage setting process (see Blanchard and Katz (1997) and (1999)). In this econ-

---

\(^1\) Aghion and Howitt (1998 pp. 127) explain the capitalization effect as follows “an increase in growth raises the rate at which the returns for creating a plant (or a firm) will grow and hence increases the capitalized value of those returns, thereby encouraging more entry by new plants and therefore more job creation”.

\(^2\) The empirical evidence about the relationship between growth and unemployment is inconclusive as one can see looking at the graph presented in Bean and Pissarides (1993). This should not be surprising because, as the result of the model presented in this and other papers, besides a possible direct relationship between both variables, there are many exogenous variables that affect them separately.

\(^3\) Gordon (1997) also argues that wage setting shocks will imply more unemployment and higher labor productivity in the short run and no long run effects.
omy, economic growth is explained by the accumulation of capital and savings decisions are modeled by using a simple overlapping generations model (OLG, henceforth).

In the model, both employment and growth are endogenous variables affected by many exogenous variables. Then, properly speaking, the positive effect of growth on employment means that the same exogenous variables affect both growth and employment. There are also some papers that analyze the effect of changes of exogenous variables on both growth and employment without a direct relationship between them.\(^4\) As an example of this literature, Daveri and Tabellini (2000), with a model similar to the one presented in this paper, show how higher labor income taxes imply less growth and less employment.\(^5\)

Our model is an AK growth model, where we assume that the technology depends on public capital, so that economic growth increases with public capital as shown by the data (see Aschauer (1989), among many others). We show that increases in public capital enhance growth, which positively affects employment when there is wage inertia.\(^6\) Recent empirical evidence has also shown that employment increases with public capital (see Pereira and Roca-Sagales (1999) and Demetriades and Mamuneas (2000)). In contrast, when the wage only depends on past wages, the employment rate grows at a constant rate until full-employment is achieved. In this case, increasing public capital enhances employment growth during the transition to full-employment. Finally, when the wage only depends on the unemployment benefit and, thus, there is no wage inertia, the employment rate is constant and does not depend on public capital. That is, in this model public capital increases growth and the effects on employment depend on the assumptions made on the wage formation process.

Summarizing, with respect to the previous literature, we provide another explanation for a positive effect of growth on employment based on wage inertia. We show that wage inertia implies permanent effects of growth on employment, contrary to Blanchard and Wolfers’ (2000) conjecture. And, by means of introducing public capital, we show that increases in taxes may have positive effects on growth and, with wage inertia, also on employment, modifying Daveri and


\(^5\)The effect of distortionary taxation in a world characterized by the presence of labor unions in an open economy is analyzed by Alesina and Perotti (1997).

\(^6\)Raurich and Sorolla (2003) present a model where public capital may increase employment in the long run when the elasticity of the labor demand with respect to wages increases with public capital. In contrast, in this paper, this elasticity is constant and public capital positively affects employment when there is wage inertia.
Tabellini’s (2000) main result.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium and shows how the properties of the equilibrium depend on the wage formation process. Also in this section, we compare the effects on growth and employment of increasing public capital when different modes of government financing are considered. Section 4 concludes.

2. The Economy

In this section we develop a simple endogenous growth model with a non-competitive labor market, that allows us to illustrate how the effects of growth on employment depend on the assumptions made on the wage formation process. We first describe the technology and the labor market. Next, we characterize the consumers’ behavior and we close the section with a description of the government budget constraint.

Firms produce the only good of the economy using the following production function introduced by Barro (1990):

\[ Y_t = AK_t^\alpha L_t^{1-\alpha} g_t^{1-\alpha}, \quad \alpha \in (0, 1), \]

where \( Y_t \) is aggregate output, \( K_t \) is the aggregate stock of capital, \( L_t \) is the labor force, and \( g_t \) measures the services derived from the stock of public capital in the economy. Profit maximization implies that factor prices are equated to marginal productivities so that the interest rate is:

\[ r_t = \frac{\partial Y_t}{\partial K_t}, \]

and the wage is

\[ w_t = \frac{\partial Y_t}{\partial L_t}. \quad (2.1) \]

Equation (2.1) characterizes the labor demand.

Following many others, we assume that the unions’ preferences are characterized by the following Stone-Geary utility function:

\[ V(w_t) = ((1 - \tau_w) (1 - \tau) w_t - w^*_t)^\gamma L_t, \quad \gamma \in (0, 1). \]

Thus, unions’ utility depends on both employment and the difference between the wage net of taxes and a reservation wage, \( w^*_t \) (see de la Croix et al. (1996)).
The parameter $\gamma$ is a measure of the concavity of the utility function with respect to the difference between the wage and the reservation wage, $\tau_w$ is a tax on the wage that employed workers pay to finance the unemployment benefit and $\tau$ is an income tax. Unions choose a wage that maximizes the utility function taking into account that the labor demand depends on the wage (union’s monopoly model). The solution of the program is

$$w_t = \frac{w_t^r}{(1 - \tau_w)(1 - \tau)(1 - \gamma \alpha)},$$

where $-\frac{1}{\alpha}$ is the elasticity of the labor demand with respect to the wage, which is constant.

Following de la Croix et al. (1996), we assume that the reservation wage is a weighted average of the unemployment benefit and of the wage in the previous period\(^7\)

$$w_t^r = \phi d_t + (1 - \phi)(1 - \tau_w)(1 - \tau)w_{t-1},$$

where $d_t$ is the unemployment benefit net of taxes and $\phi \in [0,1]$ provides a measure of the intensity of past wages in the wage formation process. It follows that the wage equation is

$$w_t = \frac{\phi d_t + (1 - \phi)(1 - \tau_w)(1 - \tau)w_{t-1}}{(1 - \tau_w)(1 - \tau)(1 - \gamma \alpha)}. \quad (2.2)$$

The previous wage equation shows the existence of wage inertia provided $\phi < 1$. Wage inertia could also be derived in an efficient wage model where workers’ disutility depends on the comparison between current and past wages (see Collard et al. (2000) and de la Croix et al. (2000)). Therefore, the assumption that drives equation (2.2) is not the wage setting under unionism but that agents’ utility depends on the comparison between present and past wages. As we have emphasized in the introduction, empirical evidence shows that there is wage inertia in the wage setting process (see Blanchard and Katz (1997) and (1999)). Finally,

\(^7\)We could also interpret $w_t^r$ as an aspiration wage. In this case the weights associated to the previous wage and the unemployment benefit would be positive constants that could be larger than one. As noted by Blanchard and Katz (1997), the reservation wage is not observable and as they say “models based on fairness suggest that the reservation wage may depend on factors such as the level and the rate of growth of wages in the past, if workers have come to consider that wage increase as fair. Perhaps a better word than reservation wage in that context is aspiration wage” (see pp. 54).
note that the labor demand, (2.1), and the wage equation, (2.2), describe the labor market.

On the consumers’ side, we consider a standard overlapping generations model (OLG, henceforth). We assume that each consumer lives for two periods. In the first period, consumers inelastically supply one unit of labor, consume, and save. In the second period, they consume the income generated by the savings accumulated during the first period. Moreover, we assume that in each period $t$, there are $N_t$ consumers in their first period of life, and that population grows at a constant growth rate, $n \geq -1$. For simplicity, we assume that consumers’ utility function is homothetic so that the savings function is a constant fraction of income, i.e., $s_t = s I_t$ where $s \in (0, 1)$, and $I_t = (1 - \tau_w) (1 - \tau) w_t$ when the consumer is employed and $I_t = d_t$ when the consumer is unemployed.\(^8\) Because each agent inelastically supplies one unit of labor in the first period, the aggregate labor supply is equal to $N_t$, and aggregate savings are equal to

$$S_t = s (L_t (1 - \tau_w) (1 - \tau) w_t + (N_t - L_t) d_t),$$

where $N_t - L_t$ are the unemployed workers that receive the unemployment benefit.

The government collects taxes in order to finance both the unemployment benefit and a public input. More precisely, the unemployment benefit, as we said, is financed by means of taxes on the wage paid by workers\(^9\), i.e.

$$(N_t - L_t) d_t = \tau_w w_t L_t, \quad (2.3)$$

and government revenues, $R_t$, are equal to

$$R_t = \tau_k r_t K_t + (1 - \tau_w) \tau w_t L_t,$$

where $\tau_k$ is the tax on the capital income. We also assume that the government devotes a fraction $v$ of the production to the public input and that the services derived from the public input are congested by the number of workers in the economy.\(^10\) This implies that the services derived from the public input are

$$g_t = \frac{v Y_t}{L_t}, \quad (2.4)$$

\(^8\)Assume that the utility function is $\ln c_1^t + \beta \ln c_2^t$, where $c_1^t$ and $c_2^{t+1}$ are consumption in the first and in the second period, respectively. Then, the fraction of income devoted to saving is $s = \frac{s c_1^t}{1 + \beta c_2^{t+1}} \in (0, 1)$.

\(^9\)De la Croix et al. (1996) assumes that the unemployment benefit is financed by means of taxes on the labor income paid by both workers and firms. For simplicity, we assume that only workers pay taxes to finance the unemployment benefit.

\(^10\)The introduction of a congestion effect avoids scale effects which are not empirically supported.
Furthermore, we assume that the government budget constraint is balanced in each period, i.e.

\[ vY_t = R_t. \]  

(2.5)

3. The Equilibrium

In this section, we derive the equations that characterize the equilibrium of this economy. To this end, we first derive the equilibrium production function and the equilibrium government budget constraint.

Substituting (2.4) into the production function and solving for \( g_t \), we obtain

\[ g_t = (vA)^\frac{1}{\alpha} \left( \frac{K_t}{L_t} \right). \]

Plugging the previous expression into the production function, we derive the production function in equilibrium\(^{11}\)

\[ Y_t = BK_t, \]

(3.1)

where \( B = A(vA)^\frac{1}{\alpha} \) measures total factor productivity. Next, combining (2.1) and (2.5), we derive the equilibrium government budget constraint

\[ v = \alpha \tau_k + (1 - \alpha) \tau (1 - \tau_w). \]

(3.2)

In equilibrium, the savings accumulated by the consumers are the next period stock of capital, i.e., \( K_{t+1} = S_t \). Using the aggregate savings function and (2.3), we get

\[ K_{t+1} = s ((1 - \tau_w) (1 - \tau) + \tau_w) w_t L_t. \]

Combining (2.1) with (3.1), we obtain the growth rate of capital

\[ \frac{K_{t+1}}{K_t} = s (1 - \alpha) B (1 - \tau (1 - \tau_w)), \]

(3.3)

\(^{11}\)The production function used by Daveri and Tabellini (2000) implies a positive effect of employment on growth. Then higher taxes imply less employment and less growth. This effect is not present in our AK production function. If we plug Daveri and Tabellini’s production function in our model then we will have a positive effect of growth on employment and of employment to growth.
which coincides with the rate of growth of output as follows from (3.1). Let us denote the economic growth rate by \( G \). Note that the economic growth rate increases with the fraction of production devoted to public capital, \( v \). Thus, public capital increases economic growth.

We proceed to obtain the equilibrium rate of employment. First, we use (2.1) and (3.1) to derive the labor demand

\[
w_t = (1 - \alpha) B \left( \frac{K_t}{L_t} \right).
\]

Using the previous equation and (3.3), we obtain

\[
\frac{w_t L_t}{w_{t-1} L_{t-1}} = G.
\]

Equation (3.4), derived from the labor demand, implies that the aggregate labor income grows at a constant growth rate which coincides with the economic growth rate. The reason is that in equilibrium the aggregate labor income is a constant fraction of production and, thus, it grows with production. The increase in the aggregate labor income may imply either larger wages or larger employment. We will show that if there is no wage inertia then an increase in economic growth fully translates into wage growth and there is no increase in the employment rate. Therefore, only when there is wage inertia, economic growth causes employment growth.

Next, we combine the wage equation, (2.2), with (2.3) to derive the growth rate of wages

\[
\frac{w_t}{w_{t-1}} = \frac{\lambda_2}{1 + n} \left( \frac{L_t}{N_t - L_t} \right),
\]

where

\[
\lambda_1 = \frac{\phi \tau_w}{(1 - \tau_w)(1 - \tau)(1 - \gamma \alpha)} \quad \text{and} \quad \lambda_2 = \frac{(1 + n)(1 - \phi)}{1 - \gamma \alpha}.
\]

Equation (3.5) shows that the growth rate of wages negatively depends on the unemployment rate. According to Blanchard and Katz (1999) this wage equation is empirically supported by US data.

Let us define the employment rate by \( l_t = \frac{L_t}{N_t} \). Combining (3.4) and (3.5), we obtain the dynamic equation that characterizes the equilibrium rate of employment

8
\begin{equation}
G = \frac{\lambda_2 \left( \frac{l_t}{l_{t-1}} \right)}{1 - \lambda_1 \left( \frac{l_t}{l_{t-1}} \right)}.
\tag{3.6}
\end{equation}

We define an equilibrium of this economy as a set of sequences \( \{l_t, K_t\}_{t=0}^\infty \) such that jointly satisfy (3.3), (3.6), an initial condition on the stock of capital, \( K_0 \), and an initial condition on the employment rate, \( l_0 \).\(^{12}\) And, we define a balanced growth path equilibrium (BGP, henceforth) as an equilibrium path where capital grows at a constant rate and the employment rate remains constant. The following proposition characterizes the BGP:

**Proposition 3.1.** There exists a unique BGP equilibrium. Along this path, \( \frac{K_t}{K_{t-1}} = G \), where

\[ G = s (1 - \alpha) B (1 - \tau (1 - \tau_w)) , \]

and

\[ l = 1 - \frac{\lambda_1}{1 + \lambda_1 - \frac{\lambda_2}{G}}. \]

**Proof.** The proof follows from (3.3) and imposing \( l_t = l \) for all \( t \) in (3.6).

In order to guarantee for a well defined BGP, that is \( l \in [0,1] \), we must assume that the parameters satisfy the following relation: \( G > \lambda_2 \). Next, in the proposition below, stability of the BGP is discussed.

**Proposition 3.2.** Assume that \( \phi \in (0,1) \). Then, the BGP equilibrium is globally stable. Thus, the dynamic equilibrium converges to the BGP from any initial condition.

**Proof.** Using (3.6), it can be shown that if \( l_t = l \) then \( \frac{\partial l_t}{\partial l_{t-1}} \in (0,1) \). This means that the BGP is locally stable and because there is a unique BGP it is also globally stable.

While the growth rate of both capital and output is constant along the equilibrium path as follows from (3.3), the employment rate changes along the transition

---

\(^{12}\)Actually, the initial condition on the employment rate follows from an initial condition on the wage, \( w_{-1} \). This is the actual initial condition because unions set the wage using the value of the wage in the previous period, as follows from (2.2). Combining this equation with (2.1), (3.1) and the initial condition on capital, the initial employment rate is obtained as a function of \( w_{-1} \).
to the BGP if $\phi \in (0, 1)$. Moreover, because $\lambda_2 > 0$ when $\phi \in (0, 1)$ and thus there is inertia in the wage formation process, the employment rate positively depends on the economic growth rate and on public capital, as follows from (3.3). This result points out the importance of the wage formation process in driving the dynamics of employment. Actually, when $\phi = 1$ and thus the reservation wage does not depend on past wages, $l_t = l = \frac{1}{1+\lambda_1}$ for all $t$. In this case, the employment rate does not exhibit transition and the growth rate does not affect the rate of employment, which implies that an increase in public capital causes more growth but does not affect the rate of employment.\textsuperscript{13} In contrast, when $\phi = 0$ and thus the reservation wage coincides with the wage in the previous period, (3.6) simplifies into the following equation:

$$\frac{l_t}{l_{t-1}} = \frac{G}{\lambda_2}.$$  

This equation defines the gross growth rate of the employment rate. Because of the assumptions made, this gross growth rate is larger than one implying that the employment rate monotonically grows until full employment is achieved. Moreover, economic growth increases the growth rate of employment during the transition.

From the previous analysis, we conclude that a permanent increase in the growth rate causes a permanent increase in the employment rate when there is wage inertia. We also conclude that the behavior of employment along the equilibrium path crucially depends on the value of the parameter $\phi$. Interestingly, the empirical literature finds that the value of $\phi$ differs substantially between countries and depending on the use of micro and macro data (see, for example, Blanchard and Katz (1997), (1999)).

In what follows we derive the long run effects on employment and on the growth rate of increasing public capital when there is wage inertia, $\phi \in (0, 1)$, and the equilibrium government budget constraint is taken into account. In other words, we compare the effects of increasing public capital under different modes of government financing and we also discuss the effects of increasing the unemployment benefit. The results are given in the following proposition:

\textsuperscript{13}The model presented in Daveri and Tabellini (2000) is this case ($\phi = 1$) without public capital. In Daveri and Tabellini model an increase in taxes imply less growth and more unemployment. Even without wage inertia ($\phi = 1$) it may be the case that in our model higher labor taxes imply less employment but higher growth due to the indirect effect of taxes on TFP via public capital. But, in this case, without wage inertia the effect of TFP on employment is switched off.
Proposition 3.3. Assume that $\phi \in (0, 1)$ and let $v^2 = (1 - \alpha) (1 - \alpha (1 - \tau_k))$, $v^1 \in (0, v^2)$, and $v^3 > v^2$. Then,

a) $\frac{\partial G}{\partial \tau_k} > 0$, and if $v > (>) v^2$ then $\frac{\partial G}{\partial \tau} < (>) 0$ and $\frac{\partial G}{\partial \tau_w} > (>) 0$.

b) $\frac{\partial l}{\partial \tau_k} > 0$, if $v < (>) v^1$ then $\frac{\partial l}{\partial \tau} > (>) 0$, and if $v < (>) v^3$ then $\frac{\partial l}{\partial \tau_w} < (>) 0$.

Proof. Part a follows from the definition of the long run growth rate in (3.3). Part b follows from the definition of the long run employment rate in Proposition 3.1.

Increasing public capital enhances the marginal product of labor and, hence, the wage increases. The increase in wages accelerates savings which explains the positive effect of public capital on economic growth. Furthermore, because of the assumptions made on the utility function, a tax on the capital income does not reduce savings. This explains the result in Part a of Proposition 3.3 that shows that increasing public capital always results in a larger growth rate, when it is financed by means of taxes on the capital income. In contrast, an increase in the tax on the labor income drives two opposite forces that affect growth. On the one hand, it increases public capital which accelerates growth. On the other hand, an increase in the tax rate on the labor income reduces income, which deters growth as savings are reduced. This explains the ambiguity on the growth effects of increasing this tax. Finally, increasing the tax that is used to finance the unemployment benefit has exactly the opposite effects. It increases the income of the people who accumulate private capital and reduces public capital. Again, this explains that the growth effects of increasing this tax rate are ambiguous and depend on the value of the fraction of production devoted to public capital.

If there is wage inertia, increasing public capital may enhance the employment rate provided it increases economic growth. When the increase in public capital is financed by means of taxes on the capital income, the employment rate always increases with this tax as growth unambiguously increases. This result is shown in Part b of Proposition 3.3. In contrast, when the increase in public capital is financed by means of taxes on the labor income, the employment rate decreases with this tax when the fraction of production devoted to public capital, $v$, is large.\footnote{Note then that if $v < v^1$ then Daveri and Tabellini’s results on the effect of labor taxes on growth and unemployment are reversed. The explanation is the following: the introduction of public capital may reverse the growth result via increase in TFP and the introduction of wage inertia may reverse the unemployment result via the effect of TFP on employment.} This negative effect may occur because taxes on the labor income enhance
the wage paid by firms and, hence, reduce employment. Finally, an increase in the tax used to finance the unemployment benefit results in a reduction in the employment rate, unless the fraction of production devoted to public capital is very large. This negative effect occurs because increasing the unemployment benefit makes the wage larger and, hence, reduces the employment rate.

4. Conclusions

In the economy developed in this paper, aggregate labor income increases together with economic growth. Thus, economic growth translates into either larger wages or larger employment. We have shown that only when there is wage inertia, economic growth does not fully translate into wage growth and, thus, causes employment growth. We have also shown that a permanent increase in the economic growth rate increases the employment rate during the transition and in the long run when there is wage inertia.

It follows that those government policies that increase growth may result in a larger employment rate. We have explored a particular fiscal policy that consists of increasing public capital and we have shown that this policy enhances growth and may increase the employment rate. While the first result does not depend on the wage formation process, the second result crucially depends on this process. This suggests the interest of further research on the actual wage equations.
References


