Abstract

Recent work on financial frictions in New Keynesian models suggest that there is a sizable spread between the risk-less interest rate and the borrowing rate. We analyze the optimal policy mix of monetary and fiscal authorities in a currency union with a country-specific credit spread by introducing a cost channel differential. The cost channel decreases the efficiency of monetary policy and increases the need for fiscal stabilization. We show that the importance of fiscal policy in stabilizing shocks increases, when there is a gap in the inflation differential due to a relative shock, an idiosyncratic shock or a credit spread differential. The welfare losses will be increasing (decreasing) in the size of the cost channel, if the nominal interest rate is a demand- (supply-) side instrument.


Keywords: cost channel; financial frictions; credit spreads; optimal monetary policy; fiscal policy; monetary union.
1 Introduction

The conventional wisdom on the optimal policy mix in a monetary union advocates monetary over fiscal policy in stabilizing the union economy (see, for example, Beetsma and Jensen, 2005, Gali and Monacelli, 2008). Such New Keynesian models typically assume that the riskless interest rate and the borrowing rate coincide. However, recent literature on the macroeconomic implications of financial frictions has shown that significant spreads do exist. This can be because of information asymmetries between lenders and borrowers, costly verification of financial contracts, bankruptcies, contagions etc. (see, e.g., Carlstrom et al., 2010; Lombardo and McAdam, 2012; Brunnermeier et al., 2013; Brzoza-Brzezina et al., 2013). Especially "tighter" financial conditions increase the size of these credit spreads which have important implications for the design and the efficiency of monetary policy. Cúrdia and Woodford (2010) and De Fiore and Tristani (2013) show that a credit spread reduces welfare, but when changes in the spread are exogenous, then the optimal target criterion remains the same as in the model without a credit spread. If there are no (for an endogenous spread: small) changes in the target criterion, the optimal monetary response to non-financial shocks takes financial frictions into account only to the extent that they affect output and inflation.

In this paper, we incorporate a country-specific spread between the riskless interest rate and the borrowing rate by introducing a cost channel differential. Using the cost channel approach is isomorphic to a model where the credit spread occurs due to a collateral constraint imposed on entrepreneurs (see Carlstrom et al., 2010). A cost channel arises when firms' marginal cost directly depends on the nominal interest rate. This may be the case as there are liquidity constraints in the factor markets or certain financial frictions. If firms need to finance their operations by borrowing funds from financial intermediaries, any change in the borrowing rate will translate into changes in the firms' marginal cost and hence their optimal price. This supply-side effect therefore lowers the efficiency of the monetary stabilization tool. Moreover, if the costs of financial intermediation are not identical across countries, the credit spreads will be country-specific and so will the pass-through of union-wide (aggregate) shocks. This heterogeneity further complicates the conduct of the interest rate policy and lowers the proficiency of the central bank to act as a shock stabilizer even more. Given the fact that there is only one monetary policy instrument in a currency union, the decline in the efficiency of monetary policy would induce large welfare losses, if the central bank was the sole policymaker responsible for stabilizing the economy. However, with national fiscal instruments available, we show that not only the decrease in monetary efficiency can be (partially) compensated, but also the cost channel differential can be treated appropriately by the use of the relative fiscal instrument. When the source of the economic disturbance is a relative or an idiosyncratic shock, the advantages of hav-
ing national instruments increase immediately as well. The flipside of the coin however, is that the use of government spending as a stabilization tool induces a welfare loss per se.

A growing number of empirical studies estimate the extent of the interest rate effect on marginal cost in the U.S. and the Euro area. Chowdhury et al. (2006) find the range of the cost channel coefficient to lie approximately between 0.2 (France), 1.3 (the U.S.) and 1.5 (Italy). This is in accordance with estimates by Ravenna and Walsh (2006) who find a cost channel of 1.276 for the U.S. Tillmann (2008) and Henzel et al. (2009) also provide supportive evidence for a significant cost channel in the Euro area. Tillmann (2009a) finds that the coefficient for the U.S. follows a U-shaped pattern. The cost channel was most important in the pre-Volcker era and less important in the Volcker-Greenspan period. De Fiore and Tristani (2013) argue that the cost channel gained quantitative importance during the recent financial crisis. From all these studies we conclude the cost channel first of all is quantitative important and secondly, that it varies across countries and over time.

As there is a manageable amount of theoretical literature on the cost channel, it is relatively straightforward to give a review in the following. Ravenna and Walsh (2006) are the first to implement the cost channel in the New Keynesian framework of a closed economy. They show that under optimal monetary policy, the output gap and inflation are allowed to fluctuate in response to both productivity and demand shocks. Tillmann (2009b) introduces uncertainty about the true size of the cost channel to the model of Ravenna and Walsh (2006). With an uncertain cost channel, the monetary authority tends to overestimate the cost-push effect of an interest rate hike which leads to a less aggressive interest rate response. Michaelis and Palek (2014) study the optimal monetary policy in a currency union with a cost channel differential and focus on demand shocks. The cost channel makes monetary policy less effective in combating inflation. On the one hand, the optimal response is a stronger use of the interest rate instrument. On the other hand, the larger the cost channel differential, the less aggressive will the optimal monetary policy be. Lam (2010), Demirel (2013) as well as Michaelis and Palek (2014) show that the value of a commitment technology of monetary policy is increasing in the size of the cost channel. Our model is closest to the framework of Michaelis and Palek (2014), but we differ by the explicit modelling of the public sector and the focus on cost-push shocks.

Since the creation of the European Monetary Union (EMU), a large number of researches investigated the role and interaction of the central bank and fiscal authorities within the currency union. One part of the literature focuses on strategic interaction between policymakers: Dixit and Lamberti (2001, 2003) study the policy mix in a game theoretical framework, when the (adhoc) objective function between the policymakers differs. Disagreement on a common objective leads to an inefficient inflation/output
outcome. Agreement on an ideal level of output and inflation leads to ideal outcomes, irrespective of which authority moves first and despite any disagreement on the relative weights of the target variables. Andersen (2005) studies the policy-mix problem when the central bank follows a strict inflation targeting policy and fiscal policymakers act strategically. He finds that there are large coordination problems with respect to aggregate shocks which increase the need of policy coordination.

Another strand of literature examines the joint optimization by monetary and fiscal authorities in the context of microfounded models with derived loss functions: Beetsma and Jensen (2005) consider the optimal policy mix between fiscal and monetary policy in a two-country version of a currency union. The roles of the policymakers are clear-cut. Monetary policy should stabilize the aggregate economy while fiscal policy ought to be utilized for stabilizing the national economies. This result is confirmed by Gali and Monacelli (2008) who study the policy mix in a currency union made up of a continuum of small open economies. Ferrero (2009) goes one step further by introducing a government budget constraint. He shows that a balanced budget rule generates welfare losses. Allowing for variations in government debt instead, is a superior policy. Kirsanova et al. (2007) also consider a government budget constraint but focus on simple fiscal policy rules rather than optimal fiscal policy. The use of fiscal policy as a stabilization tool does not harm the longer term objective of keeping public debt under control. Leith and Wren-Lewis (2011) examine the interactions between policymakers in response to shocks in government debt. When the central bank can commit, the adjustment of reducing debt is undertaken largely by the fiscal authorities. When monetary policy optimizes in a discretionary manner, reducing debts involves costly interest rate adjustments.

Until the recent financial crisis, many professional economists and policymakers neglected the role of fiscal policy as a stabilization tool. Due to concerns over persistently large budget deficits or the inflexibility in its institutional arrangement, fiscal policy was merely seen as an automatic stabilizer rather than as an active stabilization tool. However, the recent developments have led to a renewed enthusiasm for a more proactive fiscal policy (see Kirsanova and le Roux, 2013; Tulip, 2014).

In this paper we show that the optimal policy mix depends critically on the size of the cost channel (credit spread). The emergence of the cost channel makes the central bank generally less aggressive since the fiscal authority supports the monetary policymaker in stabilizing macroeconomic fluctuations. The larger the cost channel, the smaller the interest rate response and the stronger the fiscal reaction must be. Fiscal policy gains even greater importance when an inflation differential occurs due to a relative shock, an idiosyncratic shock or a cost channel differential. The welfare losses will be increasing (decreasing) by the extent of the cost channel’s strength, if the nominal interest rate is a demand- (supply-) side instrument.
The organization of the paper is as follows. In Section 2 we outline our model; the building blocks are the IS relation, the government budget constraint and the Phillips curve. Section 3 frames the joint policy problem of the monetary and fiscal authority. In Section 4, we present and discuss the inflation and output dynamics of various shocks. As our analysis will show, the nominal interest rate may turn into a supply-side instrument in the presence of a cost channel. In Section 5 we therefore discuss the determinants of this feature. Section 6 compares the inflation/output dynamics of shocks and the welfare consequences of optimal policy under discretion with optimal policy under commitment. Section 7 concludes.

2 The Model

Our model is a two-country version of a monetary union, extended to include a public sector where government spending is financed either by lump-sum or distortionary taxes. Besides the union monetary policy, the national fiscal policies act as additional stabilization tools. Hence, fiscal authorities may vary government spending when facing shocks. Goods markets are characterized by monopolistic competition and price rigidity. All goods are traded and labor serves as the only production factor. Besides these New Keynesian features, we incorporate country-specific cost channels as done by Michaelis and Palek (2014).

2.1 Optimal Consumption Choices

There is a continuum of households in the union on the interval \([0, 1]\). The population of the segment \([0, n]\) belongs to (H)ome, while the population of \([n, 1]\) belongs to (F)oreign. The representative infinitely-lived household \(j\) will seek to maximize the following objective function

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t^j)^{1-\sigma}}{1-\sigma} + \chi \frac{(G_t)^{1-\sigma}}{1-\sigma} - \frac{(L_t^j)^{1+\eta}}{1+\eta} \right],
\]

where \(\beta \in [0, 1]\) is the discount factor, \(\sigma\) is the inverse of the intertemporal elasticity of substitution, and \(\eta\) is the inverse Frisch elasticity of labor supply. \(C_t^j, G_t, L_t^j\) denote, respectively, private consumption, public consumption and hours worked while the parameter \(\chi \in [0, 1]\) measures the relative weight attached to government spending. More precisely, the private composite consumption index is defined as
\[
C^j_t = \left[ \frac{(C^j_{H,t} n (C^j_{F,t})^{1-n})}{n^n (1-n)^{1-n}} \right],
\]

where \(C^j_{H,t}\) and \(C^j_{F,t}\) are the Home and Foreign private consumption indices given by

\[
C^j_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n C^j_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{1}{1-\varepsilon}}, \quad C^j_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 C^j_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{1}{1-\varepsilon}},
\]

where \(\varepsilon > 1\) is the elasticity of substitution between any two varieties. For public consumption, we assume that the national governments only purchase goods produced in their own country. The CES functions for public consumption in countries H and F are given by

\[
G_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n G_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{1}{1-\varepsilon}}, \quad G_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 G_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{1}{1-\varepsilon}}.
\]

There are no trade barriers and preferences are assumed to be identical across countries. Therefore, absolute purchasing power parity holds and the consumer price indices are identical in H and F: \(P_t = P_t^H = P_t^F\). The consumer price index is given by \(P_t = (P_{H,t})^n (P_{F,t})^{1-n}\), where \(P_{H,t} = \left[ \left( \frac{1}{n} \right) \int_0^n (h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}\) and \(P_{F,t} = \left[ \left( \frac{1}{1-n} \right) \int_n^1 (f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}\) are the producer prices in H and F, respectively. Let us define the terms of trade as the ratio of the Foreign producer price index to the Home producer price index: \(Q_t = P_{F,t}/P_{H,t}\). Furthermore, we assume perfect consumption risk-sharing across households, both within and across countries.

The derivation of total demands for goods h and f consists of five steps: First, households divide their budget between savings and consumption. The second step implies solving the optimal allocation between Home and Foreign goods. Third, households decide on the optimal consumption choice between brands. This yields households’ demands for the individual goods, \(C^j_t(h)\) and \(C^j_t(f)\). Third, each government chooses optimally the allocation of total public consumption expenditures among varieties produced in its own country, which yields government’s demands for the individual goods, \(G_t(h)\) and \(G_t(f)\). Finally, by aggregating consumption over all households \((C^W_t = \int_0^1 C^j_t dj)\) and combining private and public demands, the total demands for goods h and f are given by

\[6\]
\[ Y_t(h) = \left( \frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} Q_t^{1-n}C_t^H + G_t(h), \quad Y_t(f) = \left( \frac{P_t(f)}{P_{F,t}} \right)^{-\varepsilon} Q_t^{-n}C_t^F + G_t(f). \] (5)

The demand for Home (Foreign) goods is increasing (decreasing) in the terms of trade which is the well-known open economy demand switching effect. Though, there is no interaction with public spending since governments consume only domestically produced goods. Using the appropriate production indices, aggregate demands can be written as

\[ Y_t^H = Q_t^{1-n}C_t^H + G_t^H, \quad Y_t^F = Q_t^{-n}C_t^F + G_t^F. \] (6)

In a cost channel environment, there are liquidity constraints for firms in the factor market. Firms have to pay for production factors before product revenues are available. Thus, it is assumed that households engage in the asset market before the goods market opens. At the beginning of period \( t \), the representative agent in country \( i \) has cash holdings \( M_{i,t-1} \), receives wage income \( W_i^i L_i^i (1-\delta_2 \tau_i^i) \) which can be used as a mean of payment and is taxed with a tax rate \( \tau_i^i \in (0,1) \) if taxation is distortionary \( (\delta_2 = 1, \text{see below}) \). This cash payoff is used for depositing funds \( D_i^i \) at financial intermediaries such that the remaining cash balances are equal to \( M_{i,t-1} + W_i^i L_i^i (1-\delta_2 \tau_i^i) - D_i^i \) when leaving the asset market. Hence, the agent faces the following cash-in-advance constraint in the goods market

\[ P_t C_t^i \leq M_{i,t-1} + W_i^i L_i^i (1-\delta_2 \tau_i^i) - D_i^i. \] (7)

At the end of the period, agents receive profit from firms \( \Pi_i^i \) and get the deposits plus interest back. Hence, the budget constraint is given by

\[ P_t C_t^i + D_i^i + M_i^i \leq R_t D_i^i + M_{i,t-1} + W_i^i L_i^i (1-\delta_2 \tau_i^i) + \Pi_i^i - \delta_1 P_i^i T_i^i - \Gamma_i^i \] (8)

where \( R_t \) is the gross nominal interest rate set by the central bank, and \( T_i^i \) and \( \Gamma_i^i \) are two types of lump-sum taxes. Let us shortly describe our modelling approach of the way taxes are levied in general. We assume that there are two possible kinds of taxes households have to pay: If taxes are collected in a lump-sum way, the dummy variable \( \delta_1 \) (\( \delta_2 \)) will be switched on (off). If taxation is distortionary, the dummy variable \( \delta_1 \) (\( \delta_2 \)) will be switched off (on).

The representative household in country \( i \) maximizes utility (1), subject to the cash-in-advance constraint (7) and the budget constraint (8). By rearranging the resulting first-order conditions, we get the Euler equation and labor supply decision respectively
Before we proceed, some simplifying notation is useful. Variables written in lower case letters denote the log of the corresponding variable (i.e., $x_t \equiv \ln X_t$), while a "^\text{\textdegree}" symbol (e.g. $\hat{x}_t \equiv \ln(X_t/X)$) is used to denote the percentage deviation of $X_t$ from its steady-state value $X$. Moreover, an aggregate (union) variable $x^w_t$ is defined as the weighted average of the national variables, $x^w_t = n x^H_t + (1-n) x^F_t$, while the relative variable $x^R_t$ is defined as $x^R_t \equiv x^H_t - x^F_t$.

The log-linearized versions of (9) can be written as

\begin{equation}
\hat{c}^W_t = E_t \hat{c}^W_{t+1} - \sigma^{-1} \left( \hat{R}_t - E_t \hat{\pi}^W_{t+1} \right)
- \delta_2 \psi_t \hat{\pi}^R_t + \hat{\pi}^I_t - \hat{\rho}_{W,t} = \eta \hat{\pi}^I_t + \sigma \hat{c}^W_t,
\end{equation}

where the aggregate producer price equals the consumer prices $\hat{p}_{W,t} \equiv n \hat{p}^H_t + (1-n) \hat{p}^F_t$, and consumer (and producer) price inflation is defined as $\pi^i_t \equiv \hat{p}_{i,t} - \hat{p}_{i,t-1}$. Furthermore, $\psi_t$ is defined as follows: $\psi_t \equiv \frac{\sigma}{1-\delta_2}$.

Finally, let us log-linearize the aggregate demand functions (6) and substitute aggregate consumption by the Euler equation (10) in order to get the national IS-curves

\begin{equation}
\hat{y}^H_t = \psi_c \left[ (1-n) q_t + E_t \hat{c}^W_{t+1} - \sigma^{-1} \left( \hat{R}_t - E_t \hat{\pi}^W_{t+1} \right) \right] + (1-\psi_c) \hat{\eta}_t + u^H_t
\end{equation}

\begin{equation}
\hat{y}^F_t = \psi_c \left[ -n q_t + E_t \hat{c}^W_{t+1} - \sigma^{-1} \left( \hat{R}_t - E_t \hat{\pi}^W_{t+1} \right) \right] + (1-\psi_c) \hat{\eta}_t + u^F_t,
\end{equation}

where $\psi_c \equiv \frac{\sigma}{1-\delta_2}$ is the private consumption share. Besides, we have added a country-specific demand shock, which is assumed to follow an AR(1) process, $u^i_t = \rho_u u^i_{t-1} + \xi^i_{u,t}$, where $\xi^i_{u,t}$ is a zero mean white noise process, and $\rho_u \in [0,1]$.

2.2 The Government Budget Constraint

The government budget constraint in country $i$ is given by

\begin{equation}
P_{t,i} G^i_t + \Phi^i W^i_t L^i_t = \delta_2 \tau^i_t W^i_t L^i_t + \delta_1 P_{t,i} T^i_t + \Gamma^i_t,
\end{equation}
where \( \Phi^i \) is a steady-state employment subsidy. We follow Rotemberg and Woodford (1997) and large parts of the literature by assuming that this steady-state subsidy is used to offset the distortions by monopolistic competition, distortionary taxation and the cost channel. We assume that the steady-state subsidy is financed by the lump-sum tax \( \Gamma^i_t \), so that \( \Phi^i W^i_t L^i_t = \Gamma^i_t \). Given this assumption, the government budget constraint can be written in real terms as

\[
G^i_t = \delta_2 \tau^i_t \frac{W^i_t}{P^i_t} L^i_t + \delta_1 T^i_t, \tag{15}
\]

and log-linearized as

\[
\hat{g}^i_t = \delta_2 \left( \tau^i_t \hat{w}^i_t - \hat{p}^i_t + \hat{\Gamma}^i_t \right) + \delta_1 \hat{T}^i_t. \tag{16}
\]

Substituting the labor supply decision (11) allows us to eliminate the nominal wage. Assuming that production is linear in labor (see (19) below), the national government budget constraints can be written as

\[
\hat{g}^H_t = \delta_2 \left( (1 + \psi) \tau^H_t + (1 + \eta) \hat{y}^H_t + \sigma \hat{c}^w_t + (1 - n) q_t \right) + \delta_1 \hat{t}^H_t, \tag{17}
\]

\[
\hat{g}^F_t = \delta_2 \left( (1 + \psi) \tau^F_t + (1 + \eta) \hat{y}^F_t + \sigma \hat{c}^w_t - n q_t \right) + \delta_1 \hat{t}^F_t. \tag{18}
\]

### 2.3 Firms

There is a continuum of firms of measure \( n \) in country \( H \) and of measure \( 1 - n \) in country \( F \). Each firm in country \( i \) produces a variety of the consumption good \( i \) using a linear technology of the form

\[
\hat{y}^i_t = \hat{\mu}^i_t. \tag{19}
\]

The adjustment of prices follows the standard treatment of staggered prices based on Calvo (1983). Only a fraction \( 1 - \theta^i \) of firms can adjust prices each period. Assuming that the steady state is characterized by zero inflation in both countries, the evolution of the producer inflation rate in region \( i \) is given by the marginal cost based (log-linearized) Phillips curve:

\[
\pi^i_t = \beta E_t \pi^i_{t+1} + \lambda^i \cdot \hat{m} c^i_t + \epsilon^i_t, \tag{20}
\]

where the composite parameter \( \lambda^i \) is given by \( \lambda^i = \frac{(1-\theta)(1-\beta^i)}{\theta} \) (see, e.g., Gali, 2008). Analogical to the assumption on the properties of the demand shock, the exogenous cost-push shock, \( \epsilon^i_t \), is assumed to be an AR(1) process, \( \epsilon^i_t = \rho \epsilon^i_{t-1} + \xi^i_{e,t} \), where \( \xi^i_{e,t} \) is a zero mean white noise process, and \( \rho, e \in [0, 1] \).

In a cost channel environment, firms have to pay the wage bill before they enter
the goods market. Households deposit funds at the beginning of the period at financial intermediaries who supply loans to firms at the nominal interest rate $R^f_t$. For simplicity we can approximate the lending rate $R^f_t$ with the policy-controlled risk-free interest rate $R_t$. Any wedge between these two interest rates will be captured by the parameter $z^i \geq 0$, which measures the strength of the country-specific cost channel. After goods have been produced and sold in the goods market, firms repay loans at the end of the period. Hence, the nominal interest rate enters the real marginal costs

$$\tilde{m}c_t^i = \tilde{w}_t - \tilde{p}_{t,t} + z^i \hat{R}_t.$$  

Real marginal costs are linear in the real wage and increasing in the nominal interest rate. Note that it is not the real interest rate which enters into the firms’ real marginal costs. The expected inflation rate does not matter since loans are assumed to be granted and repaid within a period. We proceed by eliminating the nominal wage with the help of equation (11). Then, we insert the resulting equation into the original Phillips curve (20) and use the production function (19) to derive the following price adjustment relations for Home and Foreign:

$$\pi^H_t = \beta E_t \pi^H_{t+1} + \lambda^H \left( \eta \tilde{g}_t^H + \sigma \tilde{c}_t^W + \psi_\tau \tilde{\tau}_t + (1 - n) q_t + z^H \hat{R}_t \right) + e^H_t$$ \hspace{1cm} (22)

$$\pi^F_t = \beta E_t \pi^F_{t+1} + \lambda^F \left( \eta \tilde{g}_t^F + \sigma \tilde{c}_t^W + \psi_\tau \tilde{\tau}_t - n q_t + z^F \hat{R}_t \right) + e^F_t.$$ \hspace{1cm} (23)

Finally, we substitute $\tilde{\tau}_t^i$ by the government budget constraints (17) and (18) in order to derive the national Phillips curves

$$\pi^H_t = \beta E_t \pi^H_{t+1} + \frac{\lambda^H}{1 + \delta_2 \psi_\tau} \left( (\eta - \delta_2 \psi_\tau) \tilde{g}_t^H + \sigma \tilde{c}_t^W + (1 - n) q_t + \delta_2 \psi_\tau \tilde{g}_t^H \right) + \lambda^H z^H \hat{R}_t + e^H_t$$ \hspace{1cm} (24)

$$\pi^F_t = \beta E_t \pi^F_{t+1} + \frac{\lambda^F}{1 + \delta_2 \psi_\tau} \left( (\eta - \delta_2 \psi_\tau) \tilde{g}_t^F + \sigma \tilde{c}_t^W - n q_t + \delta_2 \psi_\tau \tilde{g}_t^F \right) + \lambda^F z^F \hat{R}_t + e^F_t.$$ \hspace{1cm} (25)

Here, there are two additional arguments, $\hat{R}_t$ and, in the case of distortionary taxation ($\delta_2 = 1$), $\tilde{g}_t^i$, besides the standard features of a two-country Phillips curve. Positive government spending will have inflationary effects via the supply side, since an increase in $\tilde{g}_t^i$ has to be financed by an appropriate increase in labor income tax revenues (see (15)). This increase works analogically to a standard cost-push shock. The demand effect of an increase in government spending - aggregate demand, production, employ-
ment, wages, real marginal cost and thus prices rise - works in the same direction as long as the PC-curve is positively sloped ($\eta > \delta_2 \psi_r$), which is the case for all reasonable parameter constellations. Hence, the total effect of an increase in government spending on current inflation is definitely positive. On the contrary, if taxation is only lump-sum ($\delta_2 = 0$), there will be no supply-side effects of a rise in government spending. Then, an increase in $\hat{g}_t^i$ only shifts the IS-curve and not the Phillips curve. Thus, we can summarize that the inflationary effects of fiscal policy are larger in the case of distortionary taxation.

An increase in the central bank interest rate above its steady-state value leads to a rise in real marginal costs and thus to a rise in the current inflation rate above its steady-state value. For $H_2 > H$, the increase in $\pi_t^H$ exceeds the increase in $\pi_t^F$: a positive inflation differential $\pi_t^R \equiv \pi_t^H - \pi_t^F > 0$ emerges. The demand effect of a higher interest rate - consumption, production, employment, wages, real marginal costs and thus prices decline - works in the opposite direction. Therefore, the overall effect of a higher interest rate on current inflation is a priori ambiguous.

3 Framing the Policy Problem

In this section we describe the nature of the optimal discretionary policy and the optimal commitment policy by the monetary and fiscal authority. Following Gali and Monacelli (2008) and Beetsma and Jensen (2005), our analysis focuses on the case of full optimization, i.e. the common fiscal-monetary regime chooses jointly the union-wide nominal interest rate $\hat{R}_t$ and the size of national government spending $\hat{g}_t^i$ to maximize the utility of the representative household given by (1). This case of a centralized single policymaker corresponds to that of full coordination of monetary and fiscal authorities. Since we are interested in the output and inflation dynamics as well as the welfare losses arising from the cost channel (differential), we do not take into account any strategic interaction between both policymakers (for an analysis of this topic, see Dixit and Lambertini, 2001; 2003; among others). Instead, we are interested in the change of the optimal policy mix when monetary policy becomes less effective by the presence of a cost channel (differential).
3.1 Welfare Objective

We obtain the objective function of the single policymaker from a second-order Taylor expansion of \(1\) around the deterministic steady state (see Appendix B for details):

\[
- E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \Psi_t \right\} + o \left( \| \xi \|^3 \right),
\]

(26)

where \(o \left( \| \xi \|^3 \right)\) represents terms of order three and higher and \(\Psi_t\) denotes the per-period deadweight loss given by

\[
\Psi_t = \sigma \cdot \left( \hat{c}_t^W \right)^2 + \eta \cdot \frac{1}{\psi_c} \cdot \left[ n (\hat{y}_t^H)^2 + (1 - n) (\hat{y}_t^F)^2 \right] \\
+ \sigma \cdot \frac{1 - \psi_c}{\psi_c} \cdot \left[ n (\hat{g}_t^H)^2 + (1 - n) (\hat{g}_t^F)^2 \right] \\
+ n(1 - n) \cdot (q_t)^2 + \frac{1}{\psi_c} \cdot \left[ n \cdot \frac{\varepsilon}{\lambda_H} \cdot (\hat{\pi}_t^H)^2 + (1 - n) \cdot \frac{\varepsilon}{\lambda_F} \cdot (\hat{\pi}_t^F)^2 \right],
\]

(27)

and contains quadratic terms in \(\hat{c}_t^W, \hat{y}_t^H, \hat{y}_t^F, \hat{g}_t^H, \hat{g}_t^F, q_t, \hat{\pi}_t^H\) and \(\hat{\pi}_t^F\). The weights of the respective variables are all functions of deep model parameters. Stabilizing consumption and public spending is desirable because households are averse to private and public consumption risks. The national output gaps and the terms of trade are a part of the loss function because individuals are averse to fluctuations of hours worked and shifts of these between Home and Foreign. Inflation causes dispersion in prices and thus inefficient production of goods while differences at the national level cause undesirable relative price dispersions. As first pointed out by Benigno (2004), the country with a higher degree of price stickiness comes up with a higher degree of price distortion, and thus it is optimal to put a higher weight to the country with stickier prices. This result is replicated in (27) where the weights of the national inflation rates are increasing in the degree of price stickiness (decreasing in \(\lambda^i\)). If the duration of price contracts was identical across countries, \(\lambda^H = \lambda^F = \lambda\), the per-period loss function (27) could be rewritten in area and relative terms as
$$\Psi_t = \sigma \cdot (\hat{c}_t^W)^2 + \eta \frac{1}{\psi_c} \cdot [(\hat{g}_t^W)^2 + n(1-n) \cdot (\hat{g}_t^R)^2]$$

$$+ \sigma \frac{1-\psi_c}{\psi_c} \cdot [(\hat{g}_t^W)^2 + n(1-n) \cdot (\hat{g}_t^R)^2]$$

$$+ n(1-n) \cdot (q_t)^2 + \frac{1}{\psi_c} \frac{\varepsilon}{\lambda} \left[(\pi_t^W)^2 + n(1-n) \cdot (\pi_t^R)^2\right]. \tag{28}$$

Only then, monetary policy should stabilize $\pi_t^W$.

### 3.2 Calibration

Let us outline the parametrization for the quantitative policy analysis. The model is calibrated to a quarterly frequency and the parameters are chosen in a manner that matches the average features of countries belonging to the EMU. The discount factor $\beta$ is set equal to 0.99, so that the steady-state real interest rate is 4% p.a. By calibrating the elasticity of substitution between goods $\varepsilon$ to a value of 7.66, we assume that the steady-state mark-up of prices over marginal costs is around 15% which is a reasonable value for the European economies, according to Benigno (2004). The inverse of the intertemporal elasticity of substitution $\sigma$ is set equal to 2, following the econometric estimate of Leith and Malley (2005). Following Gali and Monacelli (2008), we assume the inverse of the Frisch elasticity of labor supply $\eta$ to be 3 and the share of public spending in GDP to be $1 - \psi_c = 0.25$, which is roughly the average for the euro zone. According to the derived steady-state relationships in Appendix A, it follows that the relative weight of government consumption in the utility function is $\chi = \left(\frac{1}{\psi_c^V} - 1\right)^{-\sigma} = \frac{1}{9}$.

The steady-state tax rate is $\tau = \frac{\psi_c^V}{(1+\psi_c^V)} = 0.2$. The price rigidity is assumed to be equal in both countries. We disregard differences in price setting between countries in order to highlight consequences from a cost channel differential. Therefore the Calvo parameter $\theta$ is set equal to a standard value of 0.75 which implies an average duration of price contracts of four quarters. We follow Benigno (2004) and Beetsma and Jensen (2005) and divide the monetary union into two equal-sized groups; thus, $n = 0.5$. For the benchmark calibration, we restrict our analysis to the case of lump-sum taxation, i.e $\delta_1 = 1$, $\delta_2 = 0$. Moreover, we adopt a degree of persistence in the shocks of $\rho_u = \rho_c = 0.5$. 

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4 Dynamics

The objective of this section is to analyze the dynamic response of the relevant endogenous variables to different kinds of supply and demand shocks. We distinguish between aggregate, asymmetric and idiosyncratic shocks. In order to avoid (too) many case differentiations, the presentation focuses on the optimal discretionary policy. For a discussion of this issue, see, e.g., Adam and Billi (2007). Let us describe our assumptions on the sequence of events before turning our focus on the inflation and output dynamics. First, the economy is in the deterministic steady state. Then, period t demand and/or cost-push shocks are revealed. Given the realizations of the shocks, the policy authority decides on the optimal response of the nominal interest rate and (national) government spending. Next, wage setters decide on the wage, and firms decide on the product price and take up a loan to finance the wage bill. Employment is pinned down, and production takes place. After selling the products on the goods market firms repay the loan.

4.1 Cost-push Shocks

4.1.1 Aggregate Cost-push Shocks

Let us first discuss the case of identical cost channels across countries, $z^H = z^F$. An aggregate cost-push shock ($e^H_t = e^F_t = e^W_t > 0$) causes union inflation to go up, whereas on the union output gap remains initially unaltered. The cost-push shock drives a wedge between the output and the inflation target, even in the absence of a cost channel. Figure 1 displays the impulse responses to a positive one percent shock in aggregate supply $e^W_t$.

Without a cost channel and for symmetric economies, we confirm the result of Beetsma and Jensen (2005), who find that the roles of monetary and fiscal policies are clear-cut: All deviations on the aggregate level will be met by the sole use of the monetary stabilization instrument, while fiscal policy remains inactive. On the contrary, all deviations of relative target variables are within the responsibility of fiscal policy, while monetary policy does not respond. Thus, in the case of the aggregate cost-push shock, the interest rate is raised by the central bank in order to mitigate the inflationary effect of the cost-push. But there will be no full accommodation. Because of a negative output gap the optimal monetary policy will tolerate an inflation rate above the target rate. Fiscal policy would be able to replicate this result, but monetary and fiscal instruments are no perfect substitutes. Any deviation of government spending from its steady-state value is, according to our welfare function (27), costly per se. Hence, the

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1See section 4.1.2 for the discussion of a relative shock.
Figure 1: Aggregate cost-push shock with identical cost channels. $z^i = 0$ (blue line), $z^i = 0.25$ (red line), $z^i = 0.5$ (green line).

monetary instrument is superior and fiscal policy remains inactive.

For $z^i > 0$, the trade-off between inflation and output worsens. A given increase in the interest rate and thus a given decline in output is now accompanied by a higher inflation rate. The cost channel makes monetary policy less effective in combating inflation, calling for actions in fiscal policy. The optimal response to the decline in monetary effectiveness is a weaker use of that instrument and a (stronger) use of the fiscal instrument. This result is in contrast to Michaelis and Palek (2014) who find that the optimal interest hike is larger instead. However, in their analysis there are no substitutes to the nominal interest rate instrument, as it is the case in our model. For a relatively small cost channel, $z^i = 0.25$, the interest hike is lower and government spending is decreased as compared to the case with no cost channel. Similar to the demand-side interest rate effect, the reduction in public spending lowers union inflation and output. For a relatively large cost channel, $z^i = 0.5$, the interest rate turns into a supply-side instrument (see Section 5). In order to reduce inflation the central bank now decreases the interest rate, as the cost channel dominates the demand channel. However, the interest rate reduction stimulates the demand for goods, which requires an even stronger reaction of fiscal policy. Regarding output and inflation dynamics, an increasing cost channel lowers the union output gap and enlarges the union inflation
Proposition 1  Suppose an aggregate cost-push shock and identical cost channels across countries. a) For \( z^i = 0 \), monetary policy is superior to fiscal policy and only an interest rate hike is used to respond to the shock. b) In the presence of a cost channel, monetary policy becomes less effective and fiscal policy steps in. Government spending is lowered to reduce the inflation gap. The optimal policy mix depends on the size of \( z^W \). The larger the cost channel, c) the smaller the interest rate response and the stronger the fiscal reaction must be; d) the smaller the union output gap and the larger the union inflation gap; e) the higher the probability of \( \hat{R}_t \) turning into a supply-side instrument. Then it is optimal to reduce the interest rate.

For symmetric countries, it is obvious that a change in a union-wide instrument, like \( \hat{R}_t \) and \( \hat{g}_t^W \), is not able to influence relative target variables. Only the relative fiscal instrument, \( \hat{g}_t^R \), can take on this role. This is no longer true in the presence of a cost channel differential since national variables matter for the conduct of monetary policy as well. The monetary authority is now able to influence both aggregate and relative variables. To illustrate the feedback on the design of the optimal policy mix we compare the scenario \( z^F = z^H = 0.25 \) with the scenario \( z^H = 0.5 \) and \( z^F = 0 \) (see Figure 2).

As the aggregate cost channel is the same for both scenarios, \( z^W = 0 \), we are able to filter the complete effect of introducing an asymmetry in the cost channel. In the case of full symmetry (blue line), all differentials are zero; all losses arise from the variability of union wide variables. In the case of a cost channel differential (red line), additional losses from deviations of relative target variables arise (see (28)). The central bank takes into account the effects of union-wide instruments on the inflation differential and the terms of trade, balancing the trade-off between a change in aggregate and relative variables. As a result, heterogeneity leads to a less aggressive monetary policy. The emergence of a cost channel differential lowers the optimal interest rate hike in response to the increase in aggregate inflation. This creates a gap in the terms of trade. Since \( z^H > z^F \), so is \( q_t = p_{F,t} - p_{H,t} < 0 \) and thus, Home faces a deterioration of its terms of trade. Demand switches from Home to Foreign and the relative output gap becomes negative. On the fiscal side, the union-wide instrument \( \hat{g}_t^W \) is used more aggressively due to the smaller interest rate response. The main fiscal tool however, is \( \hat{g}_t^R \). The fiscal authority lowers relative government spending in order to reduce relative inflation. This magnifies the drop in the relative output and the terms of trade gap. The union output gap, the consumption gap and the union inflation gap are only marginally different to the symmetry case. The main results are summarized in the following.

Proposition 2  Suppose that there is a cost-push shock and a cost channel differential with \( z^H > z^F \). The cost channel differential a) gives rise to a terms of trade gap,
Figure 2: Aggregate cost-push shock with different cost channel values. $z^i = 0.25$ (blue line); $z^H = 0.5$, $z^F = 0$ (red line).
demand switches from Home to Foreign; b) makes the optimal monetary policy less aggressive and the optimal union-wide fiscal policy more aggressive and c) lets relative fiscal policy emerge. Fiscal policy reacts stronger in that region, where the cost channel is larger.

So far we have discussed the benchmark scenario of lump-sum taxes. Next, we want to analyze the implications of distortionary taxes. When taxation is distortionary, the Phillips curve is flatter compared to the lump-sum case, i.e. \( \frac{\lambda^H}{(1+\psi^z)}(\eta-\psi_\tau) < \lambda^H \eta \), which is true for \(-1 < \eta\). A decline in the nominal interest rate, for example, increases aggregate demand via the conventional demand channel. In contrast to lump-sum taxes, the increased income will initially lead to a rise in tax revenues, if taxation is distortionary. In order to maintain the government budget constraint, the distortionary tax rate has to decline. Consequently, marginal costs decline and the Phillips curve becomes flatter. What are the implications of a flatter PC-curve in the presence of a cost channel? The slope of the Phillips curve determines the extent to which an interest rate change affects inflation via the demand channel. A unit decline in the nominal interest rate, for example, increases output by \( \frac{\psi^z}{\sigma} \) (see (12), (13)). For distortionary taxation, the demand-channel effect on inflation is \( \frac{\lambda^i}{(1+\psi^z)}(\eta-\psi_\tau) \frac{\psi^z}{\sigma} \) which is smaller than the demand-channel effect for lump-sum taxes, i.e. \( \lambda^i \eta \frac{\psi^z}{\sigma} \) as shown above. In contrast, the cost-channel effect \((-\lambda^i \psi^z\iota)\) is the same irrespective of the type of taxation. Consequently, the cost channel gains more weight - relative to the demand channel - in the case of distortionary taxation.

Again, we consider an aggregate cost-push shock which initially pushes union inflation up. In order to focus on the differences of the two tax systems for the optimal policy mix, we display only the impulse response functions of the nominal interest rate and government spending for different values of \( z^i \) in Figure 3.

As already discussed in Section 2.3, a change in government spending will change inflation in the same direction through the demand and supply side if taxation is distortionary. In other words, fiscal policy becomes more effective in closing the inflation gap. A change of the nominal interest on the contrary has opposing inflationary effects via the demand and supply side. By looking at Figure 3, we see that Proposition 1a) (blue solid line) no longer holds true. Without a cost channel, monetary policy is no longer superior at all times. Furthermore, it can be observed that government spending is decreased (blue dashed line). Fiscal policy is more effective than monetary policy in closing the inflation gap. However a policy mix is used because fiscal policy still inhabits welfare cost on its own. A second result is a stronger use of the fiscal instrument and a smaller interest rate response (dashed lines always below solid lines). Monetary policy is less aggressive as long as the nominal interest rate is a demand-side instrument, but more aggressive when it is a supply-side tool. Another feature of distortionary taxa-
Figure 3: Aggregate cost-push shock under different kinds of taxation and cost channel values. Solid lines denote lump-sum taxation; dashed lines denote distortionary taxation. $z^i = 0$ (blue line), $z^i = 0.25$ (red line), $z^i = 0.5$ (green line).

Proposition 3 Suppose an aggregate cost-push shock. Distortionary taxes, relative to lump-sum taxes, are characterized by a) a stronger use of the fiscal instrument; b) a smaller interest rate response and c) a higher probability of the nominal interest rate to turn into a supply-side instrument.

4.1.2 Relative Cost-push Shocks

We will focus on a perfect asymmetric (relative) cost-push shock, i.e. $e^H_t = -e^R_t$ ($e^R_t > 0$), whereas $e^w_t = 0$ (see Figure 4).

The consequence is initially an increase in the inflation differential, a negative terms of trade gap but a zero union inflation gap. The optimal policy mix and the impact on union inflation and union output very much depend on the sign of the cost channel differential. In the case of no or identical cost channels, $z^i = 0.25$ (blue line), a perfect
Figure 4: Relative cost-push shock with different cost channel values. $z^i = 0.25$ (blue line), $z^H = 0.5$, $z^F = 0$ (red line), $z^H = 0$, $z^F = 0.5$ (green line).

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asymmetric shock does not affect aggregate variables, \( \hat{y}_t^W = \hat{c}_t^W = \pi_t^W = 0 \). Since the aggregates are not touched and differentials cannot be influenced, the optimal monetary policy is to do nothing, which is in accordance to Lane (2000). Because of the inflation differential and the terms of trade gap, the policy authority faces a loss, but due to the assumed symmetry in the transmission process, the policy authority cannot affect country differentials by the use of a union-wide instrument such as \( \hat{R}_t \) or \( \hat{g}_t^W \). Instead, the policy authority makes use of the relative instrument by lowering relative government spending, \( \hat{g}_t^R \) and thereby reducing the inflation differential and terms of trade gap.

The optimal policy mix changes in the case of a cost channel differential. The "do nothing" result of monetary policy does not hold anymore as now the nominal interest rate affects inflation differentials and the terms of trade. From the discussion of the Home and Foreign Phillips curve (see (24) and (25)) we know that an increase in the interest rate leads to an increase (decrease) in the inflation differential for \( z^R > 0 \) (\( z^R < 0 \)). Thus, for \( z^R > 0 \) the central bank has to lower the interest rate (red line), and for \( z^R < 0 \) it has to raise the interest rate (green line). The decline in the inflation differential comes at a cost. For \( z^R > 0 \) (\( z^R < 0 \)) union output and union inflation increase (decrease). These deviations however are reduced by the union fiscal policy. For \( z^R > 0 \) (\( z^R < 0 \)) union government spending is decreased (increased).

**Proposition 4** Suppose a perfect asymmetric cost-push shock \( e_i^H = -e_i^F \) (\( e_i^R > 0 \)). In all cases, it is optimal to reduce relative government spending. The optimal policy mix depends on the cost channel differential. a) For \( z^H = z^F \), the optimal monetary policy is to do nothing. Optimal fiscal policy is restrained to the reduction of relative government spending. b) For \( z^R > 0 \), the central bank reduces the inflation differential via a lower interest rate, accepting an increase in aggregate output and aggregate inflation. This increase is dampened by a reduction of union government spending. c) For \( z^R < 0 \), the policy authority reduces the inflation differential via a higher interest rate accepting a decline in aggregate output and aggregate inflation. This decrease is mitigated by an increase of union government spending.

### 4.1.3 Idiosyncratic Cost-push Shocks

National (idiosyncratic) shocks affect both aggregate and relative supply. Take, for instance, a Home cost-push shock: \( e_i^H > 0 \) and \( e_i^F = 0 \). Such a disturbance is an aggregate cost-push shock, \( e_i^W > 0 \), as well as relative cost-push shock, \( e_i^R > 0 \). A positive inflation differential emerges, Home faces a deterioration of its terms of trade and the relative output gap becomes negative due to the decline in relative demand.

Figure 5 displays the impulse responses of the endogenous variables for identical cost channels. The results are similar to those of the aggregate cost-push shock. If there is no cost channel, \( z^i = 0 \) (blue line), the roles of monetary and fiscal policy are
Figure 5: Home cost-push shock with identical cost channels. $z^i = 0$ (blue line), $z^i = 0.25$ (red line), $z^i = 0.5$ (green line).
Figure 6: Home cost-push shock with different cost channel values. $z^i = 0.25$ (blue line), $z^H = 0.5$, $z^F = 0$ (red line), $z^H = 0$, $z^F = 0.5$ (green line).

again clear-cut. Monetary policy stabilizes the economy at the union level and fiscal policy at the national (relative) level. Therefore, the optimal policy mix consists of an increase in the interest rate, no change in union government spending and a decrease in relative government spending. The interest rate hike stabilizes aggregate inflation and output, whereas fiscal policy reduces deviations of the inflation differential as well as the terms of trade from their steady-state values.

In the presence of a cost channel, the trade-off between output and inflation stabilization worsens. There is a drop in monetary efficiency and therefore room for fiscal stabilization at the union level. For a relatively small cost channel, $z^W = 0.25$ (red line), the optimal interest rate hike is smaller and union government spending is lowered compared to the no cost channel case. For a relatively large cost channel, $z^W = 0.5$ (green line), the interest rate becomes a supply-side instrument and the optimal interest rate response is now negative. As the interest rate reduction stimulates the demand for goods, union government spending is decreased to an even greater degree. Note that the size of the cost channel does not alter the paths of all the relative variables. The relative instrument only stabilizes relative target variables and these are independent of $z^W$. The results are summarized in the following.

Proposition 5 Suppose a Home cost-push shock and identical cost channels across countries. In all cases, only relative fiscal policy reduces the inflation differential. a) Without cost channels, solely monetary policy stabilizes the economy at the union level. b) In presence of a cost channel, monetary and fiscal policy stabilize the economy at the aggregate level. The optimal policy mix depends on the size of $z^W$. The larger the cost channel, c) the smaller the interest rate response and the stronger the (union) fiscal reaction must be; d) the higher the probability of $\tilde{R}_t$ turning into a supply-side instrument.

Let us now shift our focus back to the cost channel differential. Figure 6 also displays the impulse responses to a one percent Home cost-push shock. The optimal policy strongly depends on the sign of the cost channel differential. Now the monetary authority can influence the inflation differential and the terms of trade. In all cases, a reduction in relative government spending is still the main tool for lowering the inflation differential. For $z^R = -0.5$ (green line), any increase in the interest rate reduces the inflation differential. The interest rate hike, which is also optimal in the case of identical cost channels (blue line), has a positive side effect now; it helps to reduce the inflation
differential. As a consequence, it is optimal to magnify the interest rate hike and to reduce the drop in relative government spending as compared to the case of symmetrical cost channels. As the central bank reacts more aggressively, fiscal policy will increase union government spending in order to lower the negative effects of the higher interest rate on the output gap. Relative to identical cost channels, the decline in the union output gap turns out to be magnified; the incline in the union inflation gap is smaller.

For \( z^R = 0.5 \) (red line), any increase in the interest rate widens the inflation differential. However, for this size of the cost channel differential the nominal interest rate turns into a supply-side instrument. Therefore, the central bank lowers the nominal interest rate in order to reduce union inflation. Furthermore, this policy has the positive side effect of reducing the inflation differential as well as boosting aggregate demand. Hence, the fiscal authority is less aggressive in reducing relative government spending and more aggressive in decreasing union public spending compared to the case of having symmetry in the cost channel. Following that, the decline in the union output gap turns out to be weaker; the increase in the union inflation gap turns out to be stronger.

**Proposition 6** Suppose a Home cost-push shock in the case of a cost channel differential. Relative government spending is the primary and monetary policy is the secondary tool in reducing the inflation differential. Comparing policy responses to the case of identical cost channels: a) for \( z^R < 0 \), monetary policy is more aggressive, relative public spending is less aggressive and union government spending switches the sign (to positive); b) for \( z^R > 0 \), the interest rate switches the sign (to negative) as it is a supply-side instrument, relative government spending reacts less aggressively and union public spending declines stronger.

### 4.2 Demand Shocks

In this subsection we extend our analysis to the case of aggregate and idiosyncratic demand shocks for identical cost channels. We will omit the straightforward extension of a relative demand shock or cost channel differentials, as these scenarios are greatly similar to those already discussed in the section of cost-push shocks.

#### 4.2.1 Aggregate Demand Shock

Let us discuss the case of identical cost channels across countries, \( z^H = z^F \). Figure 7 displays the impulse responses to a positive one percent shock in aggregate demand \( u_t^W \).

The demand shock creates initially inflation and a positive output gap. In absence of a cost channel, \( z^i = 0 \), our model replicates the "divine coincidence"-result of Blanchard and Gali (2007): \( \hat{y}_t^w = \pi_t^w = 0 \). An aggregate demand shock will be offset perfectly
Figure 7: Aggregate demand shock with identical cost channels. $z^i = 0$ (blue line), $z^i = 1$ (red line), $z^i = 2$ (green line).
by varying the interest rate. The interest rate necessary to bring inflation back to the target value is identical to the interest rate necessary to close the output gap. Similar to the case of cost-push shocks (see Section 4.1.1), fiscal policy would also be able to close these gaps, but again, monetary and fiscal instruments are no perfect substitutes since government spending produces a loss per se.

This solution, however, does not hold in the presence of a cost channel as it drives a wedge between the output and the inflation target. The rise of the interest rate pushes inflation up via the supply side of the economy. The cost channel makes monetary policy less effective in combating inflation so that fiscal policy comes into action as a supporting tool for stabilization. The interest hike is accompanied by a reduction of public consumption which lowers union output and inflation. The larger the cost channel the stronger the use of the fiscal instrument and the weaker the use of the monetary instrument. For a relatively small cost channel \( z^i = 1 \), the common monetary and fiscal authority accepts an increase in union inflation and the policy reaction is strong enough to turn the union output gap to negative. For a large cost channel \( z^i = 2 \), both the union output and the inflation gap remain positive which is due to two reasons. First, a larger decline in government spending could reduce \( \hat{\gamma}_t^W \) and \( \pi_t^W \), but the additional loss of a larger \( b_g W_t \)-gap outweighs. Second, a larger interest hike would suffice but the additional loss of a larger \( b_c W_t \)-gap outweighs.

**Proposition 7** Suppose identical cost channels across countries. a) The cost channel worsens the output gap/inflation trade-off and impedes the perfect neutralization of aggregate demand shocks. b) An increasing cost channel requires a stronger use of the fiscal instrument and a weaker use of the monetary instrument. c) For a large cost channel, both the inflation and the output gap are positive.

### 4.2.2 Idiosyncratic Demand Shock

An idiosyncratic shock, for example in Home demand increases aggregate demand, \( u^w_t > 0 \), as well as relative demand, \( u^R_t > 0 \). A positive output differential and a positive inflation differential emerge, therefore Home faces a deterioration of its terms of trade. Similar to the aggregate demand shock, monetary policy is able to close the union inflation and output gap by increasing the interest rate for the case of \( z^H = z^F = 0 \). Fiscal policy focuses on deviations of the relative target variables and reduces relative government spending, i.e. \( \hat{\gamma}_t^W = 0, \hat{\gamma}_t^R < 0 \). Due to the implicit welfare costs of using a fiscal instrument, the inflation differential cannot be neutralized. For \( z^H = z^F > 0 \), the cost channel drives a wedge between the target variables. As a consequence, monetary policy becomes less effective. The optimal response to the decline in effectiveness is a partial substitution of policy instruments. There is a weaker use of the monetary and a
stronger use of the fiscal union instrument. The union output gap and the consumption gap turn out to be negative; the union and relative inflation gaps are positive.

5 Monetary Policy as a Supply-side Instrument

As our analysis has shown, there are several scenarios where the nominal interest rate turns into a supply-side instrument. If that is the case, the cost channel of monetary policy will dominate the demand channel, changing the optimal policy mix significantly and has thus crucial implications for the output and inflation dynamics. In this section we lay out several factors we have identified, determining the weight of the cost channel relative to the demand channel.

The first and main argument is obvious: the strength of the cost channel itself, $z$, determines the importance of supply-side effects of monetary policy. If the cost channel was zero, there would be no supply-side effects at all and our model would collapse to a standard two-country monetary union model. The transmission mechanism of monetary policy would involve only the conventional demand channel and there would be no spread between the lending rate $R^d_t$ and the policy-controlled risk-free interest rate $R_t$. The stronger the cost channel, the higher the probability of the nominal interest rate to turn into a supply-side instrument.

Secondly, the type of taxation is of importance as we already pointed out in the discussion on aggregate cost-push shocks. In the case of distortionary taxation, the slope of the Phillips curve flattens. Hence the absolute weight of the demand channel of monetary policy weakens. As there is no direct interaction between the cost channel and the type of taxation, the relative weight of the cost channel increases (see Proposition 3c).

The third point is very similar to the previous one. Increasing the Frisch elasticity of labor supply (decreasing $\eta$) implies a relatively flat labor supply curve. A decline in the nominal interest rate, for example, shifts labor demand. The adjustment process to the new equilibrium will work rather through a change in output than a change in the nominal wage, if labor supply is relatively elastic. Thus, there is a relatively small increase in marginal costs and the Phillips curve flattens. Again, as the cost-channel effect does not change, the relative weight of the cost channel increases.

Fourthly, the private consumption share in GDP ($\psi_c$) determines the allocation of private and public consumption in aggregate demand. For a relatively small value of $\psi_c$, a given change in private consumption has only a small impact on aggregate demand. Let us underpin this case with an example: Figure 8 shows the effect of an aggregate cost-push shock for a given cost channel value of $z^f = 0.35$ but different $\psi_c$-values. We see that for the benchmark case (blue line), the nominal interest rate is a demand-
side instrument. Decreasing $\psi_c$ lowers the interest rate hike (red line) until it becomes a supply-side instrument (green line) at about $\psi_c = 0.65$, which is still a reasonable approximation of the private consumption share for the EMU-economies according to Beetsma and Jensen (2005). Regarding fiscal policy, as monetary policy becomes less aggressive when decreasing $\psi_c$, the drop in union public spending is accordingly larger. The evolution of the other endogenous variables corresponds to our analysis of the cost-push shock above.

Lastly, the intertemporal elasticity of substitution of consumption plays an important role. $\sigma$ tells the degree of intertemporal consumption smoothing. For a relative large value of $\sigma$, a given increase in the nominal interest induces only a relatively small substitution of current to future consumption. This means only a relative small shift of the IS-curve and thus an increase in the weight of the cost channel. A simulation of holding $z$ constant and increasing $\sigma$, or lowering $\eta$, would reproduce a very similar picture to Figure 8.

**Proposition 8** The probability of the nominal interest rate turning into a supply-side instrument increases a) in the size of the cost channel ($z$), b) when taxation is distortionary, c) for a decreasing private consumption share ($\psi_c$), c) for a decreasing...
intertemporal elasticity of substitution \((1/\sigma)\) and an increasing labor supply elasticity \((1/\eta)\).

6 Discretion versus Commitment

So far, our analysis considered only optimal policy under discretion. Since the terms of trade is an endogenous state variable, there is inertia in policy, even under discretion. In any stationary equilibrium, expectations about inflation will depend on the actual terms of trade, which is a function of its past values. This makes the optimization problem a dynamic one with a need to solve for a fixed point. We do so by using the algorithm provided by Dennis (2007) which makes the optimal discretionary policy then time-consistent. The limited control over inflation expectations worsens the output gap/inflation tradeoff creating the Clarida et al. (1999) stabilization bias. By worsening the output gap/inflation tradeoff even more, the cost channel is an important driver of the stabilization bias. Moreover, the stabilization bias is no longer restricted to supply shocks but also arises from demand shocks, see Demirel (2013).

If the policy authority is able to credibly commit itself to a policy plan, it is able to influence expectations systematically. The optimal policy plan takes the expectation channel into account. The policy authority optimizes over an enhanced opportunity set, so that the commitment solution must be at least as good as the one under discretion (see Sauer (2010)). The policymaker optimizes once and never reoptimizes. However, such a commitment to a history-dependent policy in the future is time-inconsistent. In any period \(t > 1\) the policy authority has an incentive to exploit expectations and to apply the same optimization procedure again. To overcome this initial-period problem, Woodford (1999) has proposed the concept of the timeless perspective. A timeless policymaker implements a policy conforming to a rule that would have been optimal to adopt in the distant past. Put differently, he promises not to exploit initial conditions. But the timeless perspective faces credibility problems too. If the economy is not close enough to its steady state, a switch from discretion to the timeless perspective can be welfare decreasing; see Sauer (2010) and Dennis (2010). In our model, the timeless perspective and the commitment solution coincide, since the initial conditions coincide (the economy starts in the deterministic steady state). Kirsanova and le Roux (2013) discuss the actual practices of different central banks and show in a model with monetary and fiscal interaction, that the past policy in the UK is better explained by optimal policy under discretion than under commitment.

\[^2\]Dennis (2007) provides a numerical procedure that solves for Markov-perfect Stackelberg-Nash equilibria. In our case, the central bank is the Stackelberg leader while the private sector is the Stackelberg follower.
Figure 9: Transitory aggregate cost-push shock under discretion (solid lines) and commitment (dashed lines). $z^i = 0$ (blue lines), $z^i = 0.25$ (red lines).

Next, we point out the differences of the optimal policy mix between discretion and commitment with and without a cost channel. We study a purely transitory cost-push shock ($\rho_e = 0$) because it illustrates the differences most clearly (see Figure 9).

In the case of discretionary policy (solid lines), all target variables ($W_t^\omega$, $\gamma_t^\omega$, $\gamma_t^\omega$, $\gamma_t^\omega$) return to their initial, zero gap, value once the shock has vanished (i.e. one period after the shock) irrespective whether there is a cost channel or not. The cost channel (red solid line) only causes a higher inflation in the initial period and requires a policy mix as stated in Proposition 1. By contrast, under the optimal policy with commitment (dashed lines), all the target variables display intrinsic persistence. The reason why the policy authority allows these deviations, which persist well beyond the life of the shock, is simple: By committing to such a response, the welfare losses, when the shock occurs ($t = 0$), are significantly lower, as the policy authority manages to improve
the trade-off between the target variables. The cost channel (red dashed line) enlarges again the wedge between output and inflation stabilization. Most importantly however, we see that the advantage of the commitment technology increases in the size of the cost channel. The decline in the gaps of the target variables due to commitment is significantly larger in presence of a cost channel. For example, the difference in union inflation between commitment and discretion amounts to approximately 0.03 without cost channel, and 0.05 with cost channel.

In table 1, we display some statistics for different cost channel calibrations under commitment (top panel) and discretion (bottom panel) in the case of a persistent positive one percent shock in aggregate supply \((\rho_\epsilon = 0.5)\). For each policy type and cost channel parametrization, table 1 shows the implied standard deviations of all the relevant target variables, expressed in percent. Moreover, the absolute welfare losses as well as the losses relative to the first-best outcome (the commitment regime) are displayed. These are expressed as a fraction of steady-state consumption that must be given up to equate welfare in the stochastic economy to that in a deterministic steady state.

Regarding the cost channel, we look at four different cases under each policy type. First, there is no cost channel; \(z^i = 0\). Second, there is a cost channel and the interest rate is a demand-side instrument; \(z^i = 0.15\). Third, there is a cost channel and the interest rate is a demand- (supply-) side instrument only under discretion (commitment); \(z^i = 0.3\). Fourth, there is a cost channel and the interest rate is a supply-side instrument; \(z^i = 0.45\).

Several results stand out. First, an increasing cost channel induces larger fluctuations in private and public consumption but a smaller variation in the output gap. Furthermore, an increasing cost channel implies larger fluctuations in inflation as long as the nominal interest rate is a demand-side instrument. When the nominal interest rate turns into a supply-side instrument, the variation in inflation is first increasing but eventually starts to decline. Which of these effects dominates the change in welfare, depends crucially on the nominal interest rate. As long as the monetary instrument is a demand-side instrument, the welfare costs are increasing by the extent of the cost channel’s strength. The opposite is true when the nominal interest rate turns into a supply-side instrument. Then, welfare losses eventually start decreasing in the size of the cost channel.

\(^3\)When evaluating the welfare losses, it is useful to know the weights of the respective target variables for the benchmark specification: \(\pi_t^W: 119; \hat{\eta}_t^W: 4; \hat{c}_t^M: 2; \hat{g}_t^W: 0.6\). It is a well-known feature of microfounded social welfare functions that the weight attached to inflation can be over twenty or more times larger than that attached to the output term (see Woodford, 2003, Ch.6). For many macroeconomists this sounds counterintuitive. There is no easy way out. Either the intuition is wrong or the model does not capture important cost drivers of the output gap. For a pragmatic view - conduct a robustness check by varying the weights - see Wren-Lewis (2011) and Kirsanova et al. (2013).
Second, the presence of a cost channel differential worsens welfare. The last two columns in each panel show the influence of a cost channel differential, when the nominal interest rate is a demand or a supply-side instrument respectively. For $z^W = 0.15$, comparing $z^R = 0$ with $z^R = 0.3$ shows that the cost channel differential increases the fluctuation in $\pi^W_t$. For $z^W = 0.45$, the comparison between $z^R = 0$ and $z^R = 0.9$ shows that the cost channel differential decreases the fluctuation in $\pi^W_t$. However, in both cases there is a drop in welfare as the trade-off between all target variables worsens.

Third, the welfare losses under commitment are always smaller than those under discretion. The ability to commit to a policy plan creates the best possible trade-off between the target variables. Hence, in the last row of table 1, we calculate the welfare losses relative to commitment. We obtain the result that the welfare loss from the inability to commit is increasing in the strength of the cost channel. Since the cost channel makes the interest rate less effective in combating inflation, the importance of the other instruments, i.e. fiscal policy and the commitment technology, immediately increases. However, as the nominal interest rate turns into a supply-side instrument, these relative welfare losses eventually start to decline again. Let us illustrate this in Figure 10, which displays the welfare losses of an aggregate cost-push shock under discretion and commitment but different realizations of the cost channel.

Figure 10 shows, that under discretion (red solid line), the welfare losses are always

<table>
<thead>
<tr>
<th>Commitment</th>
<th>$z^W (z^R)$</th>
<th>0 (0)</th>
<th>0.15 (0)</th>
<th>0.3 (0)</th>
<th>0.45 (0)</th>
<th>0.15 (0.3)</th>
<th>0.45 (0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std.dev. ($\pi^W_t$)</td>
<td>0.2210</td>
<td>0.2330</td>
<td>0.2386</td>
<td>0.2372</td>
<td>0.2331</td>
<td>0.2360</td>
<td></td>
</tr>
<tr>
<td>Std.dev. ($\bar{y}^W_t$)</td>
<td>1.9527</td>
<td>1.7282</td>
<td>1.4408</td>
<td>1.1246</td>
<td>1.7252</td>
<td>1.2683</td>
<td></td>
</tr>
<tr>
<td>Std.dev. ($\bar{c}^W_t$)</td>
<td>2.6035</td>
<td>2.9181</td>
<td>3.1391</td>
<td>3.2476</td>
<td>2.9210</td>
<td>3.1739</td>
<td></td>
</tr>
<tr>
<td>Std.dev. ($\bar{g}^W_t$)</td>
<td>0</td>
<td>0.3988</td>
<td>0.7859</td>
<td>1.1246</td>
<td>0.4033</td>
<td>0.9552</td>
<td></td>
</tr>
<tr>
<td>Loss</td>
<td>17.3097</td>
<td>17.7717</td>
<td>17.5974</td>
<td>16.8438</td>
<td>17.7730</td>
<td>17.0898</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discretion</th>
<th>$z^W (z^R)$</th>
<th>0 (0)</th>
<th>0.15 (0)</th>
<th>0.3 (0)</th>
<th>0.45 (0)</th>
<th>0.15 (0.3)</th>
<th>0.45 (0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std.dev. ($\pi^W_t$)</td>
<td>0.2729</td>
<td>0.2973</td>
<td>0.3177</td>
<td>0.3317</td>
<td>0.2978</td>
<td>0.3311</td>
<td></td>
</tr>
<tr>
<td>Std.dev. ($\bar{y}^W_t$)</td>
<td>2.0907</td>
<td>1.9734</td>
<td>1.7846</td>
<td>1.5245</td>
<td>1.9667</td>
<td>1.5385</td>
<td></td>
</tr>
<tr>
<td>Std.dev. ($\bar{c}^W_t$)</td>
<td>2.7875</td>
<td>3.0360</td>
<td>3.2447</td>
<td>3.3877</td>
<td>3.0414</td>
<td>3.3821</td>
<td></td>
</tr>
<tr>
<td>Std.dev. ($\bar{g}^W_t$)</td>
<td>0</td>
<td>0.4554</td>
<td>0.9734</td>
<td>1.5245</td>
<td>0.4715</td>
<td>1.4972</td>
<td></td>
</tr>
<tr>
<td>Loss</td>
<td>20.9439</td>
<td>22.3326</td>
<td>23.2185</td>
<td>23.4451</td>
<td>22.3584</td>
<td>23.4494</td>
<td></td>
</tr>
<tr>
<td>Relative loss</td>
<td>3.6342</td>
<td>4.5609</td>
<td>5.6211</td>
<td>6.6013</td>
<td>4.5854</td>
<td>3.3596</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Standard deviations of target variables and welfare costs of an aggregate cost-push shock
Figure 10: Absolute (red lines) and relative (blue line) welfare losses resulting from an aggregate cost-push shock under discretion (red solid line) and commitment (red dashed line) for different values of $z^W$.

higher than under commitment (red dashed line). Thus, relative welfare losses (blue solid line) are always positive. Under both policies, welfare losses are increasing in the size of the (aggregate) cost channel as long as the nominal interest rate is a demand-side instrument. Under discretion, the nominal interest rate turns into a supply-side tool around $z^W = 0.45$. Under commitment, we have a monetary supply-side instrument approximately at $z^W = 0.3$. Under both policies, the welfare losses are then eventually decreasing in the strength of the cost channel. The relative welfare losses are first rising. However, as under both policies the welfare costs are approaching zero, the relative welfare losses are approaching zero as well.

What is the reason why the welfare losses are declining in the size of the cost channel when the monetary stabilization tool is a supply-side instrument? The answer is simple: When looking at a demand shock, the best possible way to fight such a shock is using a perfect demand side tool - see the "divine coincidence"-result of Blanchard and Gali (2007). The same logic is true for a cost-push shock. The best way to fight a cost-push shock is using a perfect supply-side instrument. Normally, such a tool is not in the instrument set of the central bank and the cost-push shock cannot be perfectly stabilized. However, when the interest rate is a supply-side instrument, increasing the cost channel also increases the efficiency of the supply-side tool consequently, the welfare losses are decreasing. The main results are summarized in the following.
Proposition 9  

a) The welfare costs will be increasing in the size of the cost channel, if the nominal interest rate is a demand-side instrument. If it is a supply-side instrument, welfare losses eventually will start to decline by the extent of the cost channel’s strength. 

b) The presence of a cost channel differential worsens welfare. 

c) Commitment is always superior to discretion. 

d) The advantage of the commitment technology increases in the size of the cost channel as long as the nominal interest rate is a demand-side instrument. 

7 Conclusions 

This paper investigates the joint conduct of optimal monetary and fiscal policy in a currency union, when there is a country-specific wedge between the riskless interest rate and the borrowing rate. This spread is captured by the cost channel coefficient. We show that without a cost channel, there is a clear-cut solution for the assignment problem of policy instruments. Monetary policy stabilizes the economy at the union level; fiscal policy stabilizes national economies. In presence of a cost channel, monetary policy becomes less effective and fiscal policy comes into action. The optimal policy mix depends on the strength of the cost channel: The larger the cost channel, the smaller the interest rate response and the stronger the fiscal reaction must be. The emergence of an inflation differential, due to a relative shock, an idiosyncratic shock or a cost channel differential strengthens the role of fiscal policy in the stabilization process. Further, we show that in presence of a cost channel, the nominal interest rate may turn into a supply-side instrument due to a various number of reasons. Finally, we compare the optimal policy mix under discretion with the optimal policy under commitment. It is noticeable that commitment is always superior to discretion. The welfare losses will be increasing (decreasing) in the strength of the cost channel, if the nominal interest rate is a demand- (supply-) side instrument. 

Appendix 

Appendix A: The Social Planner’s Problem 

Here, we derive the social planner’s problem, which is mainly used to show that the steady state is efficient and it defines the steady-state parameters contained in the log-linearized model for a given $\frac{G}{Y}$ ratio. The social planner maximizes the welfare of a representative household, which is a weighted average of the Home’s and Foreign’s utility function (1), subject to the technology constraint (19) and aggregate demands in H and F (6). The Lagrangian in any given period can be written as
This yields the following FOCs in steady state:

\[
\begin{align*}
\frac{\partial L_t}{\partial L} &= \frac{\partial C_t}{\partial C} + \frac{\partial G_t}{\partial G} + \frac{\partial L_t}{\partial L} - \frac{\partial \bar{Y}}{\partial \bar{Y}}; \\
\frac{\partial L_t}{\partial \bar{Y}} &= \frac{\partial C_t}{\partial \bar{Y}} + \frac{\partial G_t}{\partial \bar{Y}} + \frac{\partial L_t}{\partial \bar{Y}} - \frac{\partial \bar{Y}}{\partial \bar{Y}}.
\end{align*}
\]

Note that \( C_H = C_F = C; G_H = G_F = G; L_H = L_F = L = Y \) because we assume perfect risk sharing and symmetry in steady state. Recalling the resource constraint in steady state, \( \bar{Y} = \bar{C} + \bar{G} \), the efficient steady-state output is given by

\[
\bar{Y}^* = \left(1 + \frac{1}{\delta} \right) \frac{\bar{C}}{\bar{Y}},
\]

where the superscript "*" marks the efficient level of the corresponding variable. In order for this level output to be achieved, a steady-state government spending rule must be chosen accordingly

\[
\frac{\bar{G}}{\bar{Y}} = \left(1 + \frac{1}{\delta} \right)^{-1}.
\]

For a given \( \frac{\bar{G}}{\bar{Y}} \) ratio, we can define the steady-state real wage and the steady-state tax rate. Optimal labor supply implies that \( W = \frac{\bar{W}}{\bar{P}} (1 - \delta_2 \bar{T}) \). Hence,

\[
\frac{\bar{W}}{\bar{P}} = \frac{1}{(1 - \delta_2 \bar{T})}.
\]

The steady-state government budget constraint is given by \( \bar{G} = \delta_2 \bar{T} \frac{\bar{W}}{\bar{P}} \bar{L} + \delta_1 \bar{T} \). The tax rate for lump-sum and distortionary taxation respectively, is given by

\[
\bar{T} = \bar{G} / \bar{Y}, \quad \bar{T} = \frac{\bar{G}}{(1 + \bar{G} / \bar{Y})}.
\]

Next, we show that the steady state is efficient when the steady-state subsidy, \( \Phi^* \), is chosen optimally. Note that flexible prices imply that steady-state real marginal costs
can be written as $mc^i = \frac{\varepsilon - 1}{\varepsilon (1 - \Phi^i)} \frac{1}{\Phi^i}$. Cost minimization implies that $mc^i = \frac{W}{P}$. Recall that in steady state $Q = \frac{P}{P'} = 1$ and $P' = (P_{H'})^n = \frac{P}{1 - n}$. It follows that $\frac{\varepsilon - 1}{\varepsilon (1 - \Phi^i)} = \frac{W}{P}$. This relation can be used to eliminate the real wage in the optimal labor supply decision

$$L'^i \sigma = \frac{\varepsilon - 1}{\varepsilon (1 - \Phi^i)} \frac{1}{\Phi^i}.$$  

(A.7)

Therefore, the steady-state subsidy needed to offset any distortions is given by

$$1 - \Phi^i = \frac{\varepsilon - 1}{\varepsilon (1 - \delta_2 \Phi^i)} \frac{1}{\Phi^i}.$$  

(A.8)

If this steady-state subsidy is in place, then $L'^i = C^{-\sigma}$, which corresponds to the social planner’s outcome.

**Appendix B: Union’s Welfare Loss**

The central bank’s loss function is given by

$$\Lambda_t = U(C^W_t) + nU(G^H_t) + (1 - n)U(G^F_t) - nV(L^H_t) - (1 - n)V(L^F_t).$$  

(B.1)

Subtracting the corresponding steady-state values, this can be rewritten as

$$\Lambda_t - \Lambda = U(C^W_t) - U(C) + n[U(G^H_t) - U(G)] + (1 - n)U(G^F_t) - U(G)$$

$$- n[V(L^H_t) - V(L)] - (1 - n)V(L^F_t) - V(L).$$  

(B.2)

A second-order approximation of the consumption part in the utility function \(\Pi\), $U(C^W_t)$ around its steady-state value $C$ yields

$$U(C^W_t) - U(C) = C^{1-\sigma} \left[ \bar{c}^W_t + \frac{1 - \sigma}{2} (\bar{c}^W_t)^2 \right] + o (|| \xi ||^3).$$  

(B.3)

For country $i$, taking a second-order approximation of the government spending term around its steady-state value yields

$$U(G^i_t) - U(G) = C^{1-\sigma} \left[ \bar{g}^i_t + \frac{1 - \sigma}{2} (\bar{g}^i_t)^2 \right] + o (|| \xi ||^3).$$  

(B.4)
Taking a second-order expansion of the labor supply term, we get

\[ V(L_t) - V( \bar{L} ) = L^{1+\eta} \left[ \hat{\tilde{L}}^t + \frac{1 + \eta}{2} (\hat{\tilde{L}}^t)^2 \right] + o \left( \| \xi \|^3 \right). \]  

(B.5)

Now, we want to relate employment to the output gap. Combining the production function with the total demand for \( h \) yields

\[ L^H_t = \left( Q_t^{1-n} C_t^W + G_t^H \right) \int_0^\eta \left( \frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} dh \]

\[ = Y_t^H \int_0^\eta \left( \frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} dh. \]  

(B.6)

In terms of log deviations

\[ \hat{\hat{l}}^H_t = \hat{y}_t^H + \ln \int_0^\eta \left( \frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} dh. \]  

(B.7)

It can be shown (see Gali 2008) that

\[ \ln \int_0^\eta \left( \frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} dh = \frac{\varepsilon}{2} \text{var}_h p_t(h) + o \left( \| \xi \|^3 \right). \]  

(B.8)

Hence

\[ \hat{\hat{l}}^H_t = \hat{y}_t^H + \frac{\varepsilon}{2} \text{var}_h p_t(h) + o \left( \| \xi \|^3 \right). \]  

(B.9)

Similarly, the Foreign labor supply gap can be stated as

\[ \hat{\hat{l}}^F_t = \hat{y}_t^F + \frac{\varepsilon}{2} \text{var}_f p_t(f) + o \left( \| \xi \|^3 \right). \]  

(B.10)

These expressions can be inserted in (B.5) which yields

\[ V(L^H_t) - V( \bar{L} ) = L^{1+\eta} \left[ \hat{y}_t^H + \frac{1 + \eta}{2} (\hat{y}_t^H)^2 + \frac{\varepsilon}{2} \text{var}_h p_t(h) \right] + o \left( \| \xi \|^3 \right) \]  

(B.11)
for Home and

$$V(L_t^F) - V(L) = \mathcal{L}^{1+\eta} \left[ \hat{g}_t^F + \frac{1 + \eta}{2} \left( \hat{g}_t^F \right)^2 + \frac{\varepsilon}{2} \text{var}_F p_t(f) \right] + o \left( \| \xi \| \right)^3 \quad (B.12)$$

for Foreign.

Now, we insert (B.3), (B.4), (B.11) and (B.12) in (B.2) and write the loss function as

$$\Lambda_t - \bar{\lambda} = \mathcal{C}^{1-\sigma} \left[ \hat{c}_t^W + \frac{1 - \sigma}{2} \left( \hat{c}_t^W \right)^2 \right] + n\chi \mathcal{G}^{1-\sigma} \left[ \hat{g}_t^H + \frac{1 - \sigma}{2} \left( \hat{g}_t^H \right)^2 \right]$$

$$+ (1 - n)\chi \mathcal{G}^{1-\sigma} \left[ \hat{g}_t^F + \frac{1 - \sigma}{2} \left( \hat{g}_t^F \right)^2 \right]$$

$$- n\mathcal{L}^{1+\eta} \left[ \hat{y}_t^H + \frac{1 + \eta}{2} \left( \hat{y}_t^H \right)^2 + \frac{\varepsilon}{2} \text{var}_H p_t(h) \right]$$

$$- (1 - n)\mathcal{L}^{1+\eta} \left[ \hat{y}_t^F + \frac{1 + \eta}{2} \left( \hat{y}_t^F \right)^2 + \frac{\varepsilon}{2} \text{var}_F p_t(f) \right] + o \left( \| \xi \| \right)^3 \quad (B.13)$$

From the social planner (see Appendix A) we know that the steady state will be efficient if the steady-state subsidy is chosen optimally (A.8). Then

$$\mathcal{C}^{1-\sigma} \frac{\hat{y}}{c} = \mathcal{L}^{1+\eta}$$

and

$$\mathcal{C}^{1-\sigma} \frac{1}{\psi_c} = \mathcal{L}^{1+\eta} \quad (B.14)$$

and

$$\mathcal{C}^{1-\sigma} \frac{\hat{G}}{\hat{C}} = \chi \mathcal{G}^{1-\sigma}$$

$$\mathcal{C}^{1-\sigma} \left( \frac{1 - \psi_c}{\psi_c} \right) = \chi \mathcal{G}^{1-\sigma} \quad (B.15)$$
By inserting (B.14) and (B.15), we can rewrite (B.13) as

\[ \Lambda_t - \overline{\Lambda} = C^{1-\sigma} \left[ \frac{1}{2} \left( \tilde{c}_t^W + \frac{1}{2} \psi_c (\tilde{c}_t^W)^2 \right) \right] + nC^{1-\sigma} \left( \frac{1}{\psi_c} \tilde{g}_t^H + \frac{1}{2} \left( \tilde{g}_t^H \right)^2 \right) \\
+ (1-n)C^{1-\sigma} \left( \frac{1}{\psi_c} \tilde{g}_t^F + \frac{1}{2} \left( \tilde{g}_t^F \right)^2 \right) \\
- nC^{1-\sigma} \frac{1}{\psi_c} \left[ \tilde{g}_t^H + \frac{1+\eta}{2} \left( \tilde{g}_t^H \right)^2 + \frac{\varepsilon}{2} \text{var}_{h}p_t(h) \right] \\
- (1-n)C^{1-\sigma} \frac{1}{\psi_c} \left[ \tilde{g}_t^F + \frac{1+\eta}{2} \left( \tilde{g}_t^F \right)^2 + \frac{\varepsilon}{2} \text{var}_{f}p_t(f) \right] + o(\| \xi \|^3) \]

or

\[ \frac{\Lambda_t - \overline{\Lambda}}{UC\overline{C}} = \frac{1}{2} \left( \frac{1}{\psi_c} \tilde{c}_t^W \right)^2 + n \left( \frac{1}{\psi_c} \tilde{g}_t^H + \frac{1}{2} \left( \tilde{g}_t^H \right)^2 \right) \\
+ (1-n) \left( \frac{1}{\psi_c} \tilde{g}_t^F + \frac{1}{2} \left( \tilde{g}_t^F \right)^2 \right) \\
- n \frac{1}{\psi_c} \left[ \tilde{g}_t^H + \frac{1+\eta}{2} \left( \tilde{g}_t^H \right)^2 + \frac{\varepsilon}{2} \text{var}_{h}p_t(h) \right] \\
- (1-n) \frac{1}{\psi_c} \left[ \tilde{g}_t^F + \frac{1+\eta}{2} \left( \tilde{g}_t^F \right)^2 + \frac{\varepsilon}{2} \text{var}_{f}p_t(f) \right] + o(\| \xi \|^3). \quad \text{(B.16)} \]

Before we continue, we perform a second-order expansion of aggregate demands (6) in order to eliminate all non-quadratic terms in the loss function:

\[ \tilde{g}_t^H + \frac{1}{2} \left( \tilde{g}_t^H \right)^2 = \psi_c (1-n)q_t + \psi_c \tilde{c}_t^W + \frac{1}{2} \psi_c (1-n)^2 q_t^2 + \frac{1}{2} \psi_c \tilde{c}_t^W + \psi_c (1-n)q_t \tilde{c}_t^W \\
+(1-\psi_c) \tilde{g}_t^H + \frac{1}{2} (1-\psi_c) \left( \tilde{g}_t^H \right)^2 + o(\| \xi \|^3) \]

\[ \tilde{g}_t^F + \frac{1}{2} \left( \tilde{g}_t^F \right)^2 = -\psi_c n q_t + \psi_c \tilde{c}_t^W + \frac{1}{2} \psi_c n q_t^2 + \frac{1}{2} \psi_c \tilde{c}_t^W + \psi_c n q_t \tilde{c}_t^W \\
+(1-\psi_c) \tilde{g}_t^F + \frac{1}{2} (1-\psi_c) \left( \tilde{g}_t^F \right)^2 + o(\| \xi \|^3). \quad \text{(B.17)} \]

Using these expansions, the loss function (B.16) can be written as
Finally, the union’s welfare function is given by

\[
W = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\Lambda_t - \bar{\Lambda}}{UC\bar{C}} \right] + o\left(\|\xi\|^3\right)
\]

It can be shown (see Woodford, 2003, chap. 6) that

\[
\sum_{t=0}^{\infty} \beta^t \text{var}_i p_t(i) = \frac{\beta^i}{(1 - \theta^i)(1 - \beta \theta^i)} \sum_{t=0}^{\infty} \beta^t (\pi_i^t)^2.
\]

Finally, the union’s welfare function is given by

\[
W = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\Lambda_t - \bar{\Lambda}}{UC\bar{C}} \right] = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \Psi_t + o\left(\|\xi\|^3\right)
\]

We can simplify this as

\[
\frac{\Lambda_t - \bar{\Lambda}}{UC\bar{C}} = -\frac{\sigma}{2} (\bar{c}_t^W)^2 - \frac{n}{2} \left( \frac{1}{\psi_c} \right) (\bar{g}_t^H)^2 - \frac{\sigma}{2} (1 - n) \left( \frac{1}{\psi_c} \right) (\bar{g}_t^F)^2
\]

\[
-\frac{n}{2} \frac{1}{\psi_c} (\bar{g}_t^H)^2 - \frac{n}{2} (1 - n) \frac{1}{\psi_c} (\bar{g}_t^F)^2 - \frac{1}{2} n (1 - n) q_t^2 - \frac{1}{\psi_c} \text{var}_h p_t(h)
\]

\[
-(1 - n) \frac{\varepsilon}{\psi_c^2} \text{var}_f p_t(f) + o\left(\|\xi\|^3\right).
\]
References


